Electron Cooling for a Muon Collider

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Electron cooling of muons should enable an enormous increase in muon beam phase space density. An evaluation of the cooling process indicates that the muon phase space can be reduced by a factor of about one hundred billion during several microseconds of cooling time. This analysis indicates that electron cooling of muons will enable future high energy physics colliders to be far less costly than contemporary alternatives.

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The advance of mankind's knowledge about the ultimate structure of the universe has always involved experimentation, and while alchemy and chemistry led to important early discoveries, it was the advent of particle beam experimentation that enabled the most rapid unveiling of nature's secrets. By scattering particles off of target materials, the resulting collision remnants could be studied to determine what makes up our world. Such experimentation began with cathode ray tubes, which were quickly followed by studies using electrostatically accelerated ion beams. Electrostatic accelerators gave way to circular and linear accelerators to achieve ever higher beam energies. When relativistic effects caused great difficulty in increasing the center of mass energy, physicists turned toward the use of colliding beams for even greater probing power.

Presently, new limitations inhibit the advance of our knowledge. The stable particles used for experimentation, which include electrons, ions and their antimatter partners, each have problematic issues associated with their use. Ions, even the most simplest (the proton), are composite particles. At the highest energies of today's experiments, the composite nature of ions leads to extremely complex experimental results, since each component of the ion can interact with each component of its counterpart over a wide range of possible momentum transfers during the collision event. Electrons and positrons, on the other hand, radiate photons when accelerated. Electrons will radiate these photons when they are accelerated transversely to contain them in a storage ring, which presently limits the usefulness of electron and positron storage rings. Perhaps more problematically, electrons and positrons will radiate photons when accelerated due to the fields of an oncoming colliding beam. This effect, the beamstrahlung effect[1,2,3], severely limits the usefulness of future colliding beam experiments involving electrons and positrons as the colliding particles.

With the present problems inherent in electron and ion beam accelerators, interest has turned to the use of muons in colliding beam experiments[4,5,6,7,8]. Since the muon is heavier than the electron by a factor of about 200, and since the radiative power scales strongly with the inverse of mass, muons will not have the circular acceleration and beamstrahlung radiative problems present in electron beams. Furthermore, since the muon is presently believed to be an elementary particle with no internal constituents, muon colliders will lead to the clean experimental results that are ideal for high energy physics experimentation.

A muon collider therefore has clearly desirable qualities, but it also presents two difficulties. The first problem is the fact that muons have a half life of just 2.2 microseconds, meaning that the muons must be produced, formed into beams, accelerated and collided very quickly in order to obtain useful experimental data. The second problem is that production of muons by colliding protons with fixed targets leads to a very large phase space area for the muon beam. In fact, the phase space is so large that useful beams are not possible unless a very large phase space reduction can be achieved. Studies over the past several years have shown how some phase space reduction can be achieved, but these studies have failed to arrive at a design with appropriate phase space reduction for a serious high energy physics device.

Electron cooling is a well known phase space reduction technique that was proposed by Budker in 1966[9] based on work done by Spitzer[10]. Spitzer showed how warm ions come to thermal equilibrium with electrons in a plasma, and Budker realized that an electron beam is simply a moving electron plasma. Hence, by superimposing an electron beam on a co-moving particle beam, particles can have their phase space area reduced by collisions between the particles and the electrons.

Standard equations for electron cooling times are:

(for dominant longitudinal velocity) $t_{cool} = (dp/p)^3 a^2 e \beta^4 \gamma^2 / [12 I r_e r_i \ln(B)]$. (1)

(for dominant transverse velocity) $t_{cool} = \theta^3 a^2 e \beta^4 \gamma^2 / [12 I r_e r_i \ln(B)]$. (2)

In Equations (1) and (2) dp/p is the momentum deviation and θ is the angular deviation of the particle being cooled, a is the radius of the overlapped beams, e is the charge on the electron ($1.602\text{x}10^{-19}$ C), I is the cooling beam current, r_e is the classical radius of the electron ($r_e = 2.82\text{x}10^{-13}$ cm), r_i is the classical radius of the particle being cooled, (which are muons in this case $r_i = 1.36\text{x}10^{-15}$ cm), ln(B) is the Coulomb log (approximately 10 for typical electron coolers) β is the velocity of the beams divided by the velocity of light and $\gamma = (1-\beta^2)^{-1/2}$. (Note that it is usual to find a factor of C/L in electron cooling time expressions, where C is the circumference of the ring and L is the length of the cooling section, but for muon cooling it will be desired to cool in a linear single pass so this factor reduces to unity here.) Combining the constants we have $e/[120 r_e r_i] = 3.48\text{x}10^6$ C/cm^2. Lastly, note that we will want to choose conditions where θ = (dp/p), so that cooling is not dominated by one velocity component. This leaves the cooling expression as:

$t_{cool} = \theta^3 a^2 e \beta^4 \gamma^2 / [120 I r_e r_i] = 3.48\text{x}10^6 \text{x} \theta^3 \beta^4 \gamma^2 / (I/a^2)$ (with I/a^2 in A/cm^2). (3)

One set of parameters that yields a desired cooling time is: θ = (dp/p) = 24 mRad; $\beta = 0.03$, $\gamma = 1.00045$ and $I/\pi a^2 = 10$ A/cm^2, or $I/a^2 = 31.4$ A/cm^2. (Note that presently available thermionic cathodes can readily produce $I/\pi a^2 = 10$ A/cm^2.) For these parameters, $t_{cool} = \theta^3 a^2 e \beta^4 \gamma^2 / [120 I r_e r_i]$ = 1.24 microseconds.

In principle, any values of θ and dp/p can be obtained at the entry to the cooler from any source emittance, since the invariant quantity is the beam phase space, which is $\pi r \theta$ for the transverse and $\pi dt{*}dE$ for the longitudinal. Hence, by adjusting r and dt, one can arrive at the desired values of θ and dp/p. (dp/p is related to dE.) The relevant issue is whether the values of r and dt so obtained are acceptable. For this letter, I will explore what values of r and dt are needed assuming that we use the output of a ring cooler as the input values for the electron cooler. One ring cooler design[11] predicts one sigma emittances after l5 turns of $\varepsilon_{nx} = 0.51$ cm, $\varepsilon_{ny} = 0.36$ cm, and $\varepsilon_{nz} = 0.81$ cm. The result is obtained using the ICOOL program[12], which defines the emittances as $\varepsilon_{nx} = \beta\gamma\sigma_x\sigma_{\theta x}$, $\varepsilon_{ny} = \beta\gamma\sigma_y\sigma_{\theta y}$ and $\varepsilon_{nz} = \beta\gamma\sigma_z\sigma_p/p$. As discussed further down, it will be assumed that we only wish to cool particles in approximately the inner half sigma of the distribution, and hence here we will investigate cooling initial emittances $\varepsilon_{nx} = \varepsilon_{ny} = 0.25$ cm and $\varepsilon_{nz} = 0.4$ cm. This leaves:

$\sigma_x = \sigma_y = (0.25 \text{ cm})/(0.024\text{x}0.03) = 347.2 \text{ cm} = 3.472 \text{ m}$, (4)

$\sigma_t = \sigma_z/\beta c = (0.4\ \text{cm})/(0.024\text{x}0.03\text{x}0.03\text{x}3\text{x}10^{10}\text{cm/s}) = 0.617$ microseconds. (5)

(Both σ_t and σ_x are acceptable for use in a muon collider source.)

Unsurprisingly, the required cooler is quite large. While one could naively consider building a single electron cooling beam with a 3.472 m radius, such a beam would have a cross sectional area of 378,700 cm^2 and hence need an electron beam current of 3.787 MA. Self magnetic field effects are one of many possible issues of such a device. However, note that one could instead use many smaller coolers in parallel, using upstream magnets to filament, divide and steer the muon beam filaments into the individual, parallel, coolers. (See Figures 1 and 2.) Note that while a significant loss of muons may occur in the filamentation process, such losses will be acceptable in comparison to the phase space density increase predicted below. One possible configuration would be to use square, 10 cm by 10 cm cathodes to generate the electron beams. For such a configuration, one would need about 3800 electron beam systems. Forming the system 20 layers high would require that each layer contain 190 systems, leading to an acceptable system width.

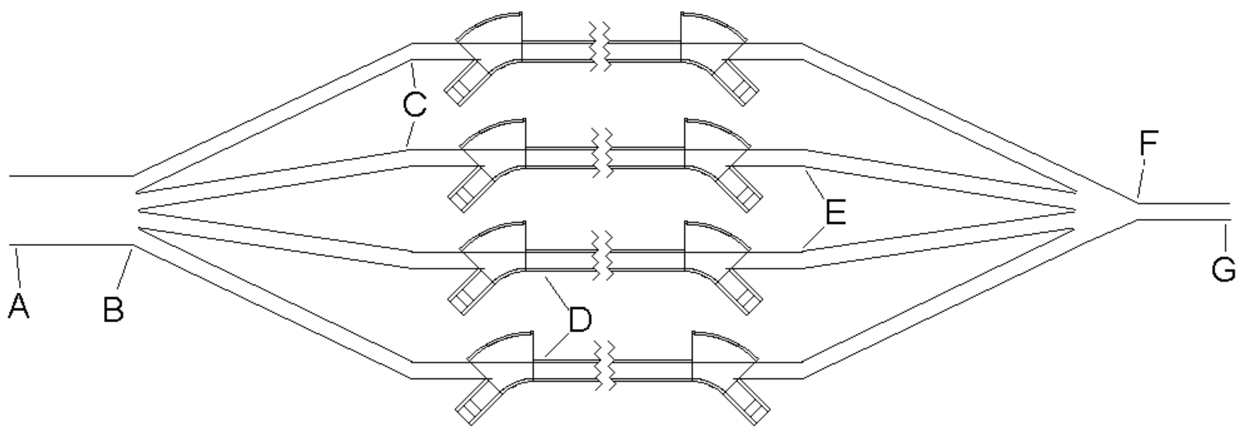

Figure 1. Filamentation Electron Cooling of a Muon Beam. A) muon beam leaves ring cooler as one large continuous beam. B) Dipoles then bend portions of the beam away from each other forming filaments. C) Downstream dipoles then bend the filaments back so that they are parallel to each other. D) Filaments are cooled in electron coolers. E) When cooling is complete, dipoles bend filaments toward a coalescing plane. F) Dipoles coalesce the muon filaments. G) A single, cooled muon beam. The process will be done in both dimensions perpendicular to beam motion; only one of which is shown here.

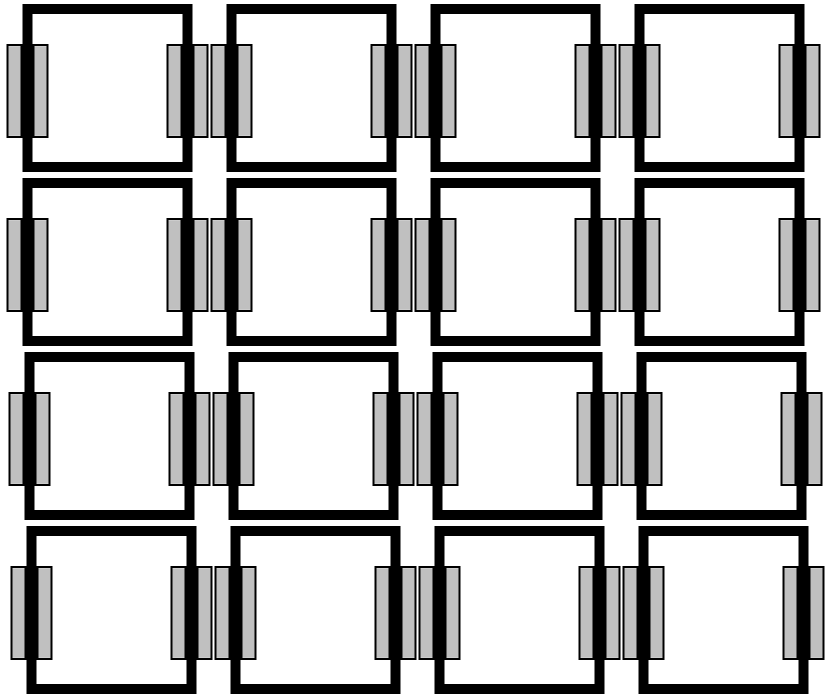

Figure 2. Square dipole septa design that could be used for separation (as looking into the direction of muon beam motion). Black indicates ferrous metal; grey indicates copper current carrying wires. Each such dipole can have a different field, allowing controlled separation of individual filaments. Dipoles can be rotated from what is shown here – this diagram is simply meant to convey the idea in a simple fashion.

Electron cooling can be very powerful once beams are sufficiently already cool enough to take advantage of this power. Looking again at Equation 3, $t_{cool} = 3.48x10^6 x\theta^3\beta^4\gamma^2/(I/a^2)$, it can be seen that the electron cooling time scales as the third power of the angle of the particle trajectories to be cooled, and the fourth power of β. A first useful observation employed here is the fact that electron cooling depends upon only one variable in phase space – the velocity – and therefore by manipulating the phase space ellipse one can use electron cooling in very advantageous ways. A second useful observation is that electron cooling is extremely powerful at very low beam energies, and that observation led to a design where the beam velocity is set at 0.03c. A third useful observation is that we wish to cool only those particles that can be cooled quickly, so as not to waste time. Particles in the tail of the distribution will take too long to cool, during which time other particles will decay away. This is the reason for only cooling one half sigma in each dimension – the cooling time is eight times less than it would be if we attempted to cool a full sigma, yet the number of particles cooled in that time is considerably more than one eighth. For a muon collider, an extremely important consideration is time, and it does not make sense to pursue cooling beyond the point where losses exceed the gains.

For non-magnetized cooling, thermal equilibrium occurs when $\frac{1}{2}m_e\sigma_{ve}^2 = \frac{1}{2}m_\mu\sigma_{v\mu}^2$. (Magnetized cooling improves this; here the simpler more conservative theory is employed.) For the electrons, which emanate from a hot cathode, $\frac{1}{2}m_e\sigma_{ve}^2 = 0.1$ eV, or $\sigma_{ve} = 6.26\text{x}10^{-4}c$. The thermal angle of the electrons is hence $\theta_e = \sigma_{ve}/\beta c = 0.0209$, and the muons will cool to the square root of the mass ratio less than this, or $\theta_\mu = 1.45\text{x}10^{-3}$, which is a factor of 16.6 less than the value at the entry point to the cooler, meaning the transverse emittances would be reduced from 0.25 cm (half sigma) to $1.51\text{x}10^{-2}$ cm (full sigma) within the cooler.

Once the muons exit the cooler just described, they should be focused through a waist and made parallel again at a smaller beam radius to again arrange for a thermal angle of about 24 mrad. The muons can then be cooled in a second stage cooler, reducing each emittance by another factor of 16.6. The process can be repeated in a third stage to get yet another reduction of 16.6. After three stages of electron cooling, the transverse emittances will be reduced by a factor of $(16.6)^3 = 4574$, to 0.5 mm-mr, which is close to ideal for a muon collider. (One could consider going lower with yet another stage, but then difficulties in steering the final beam to the resulting small spot, as well as instabilities, might begin to appear.) In the longitudinal phase space plane similar manipulations should be used. The beam should have a relatively large bunch length in the first stage and smaller bunch lengths in succeeding stages in order to cool dp/p in each stage. Longitudinally, the momentum spread of the electron beam is given by $dp/p = \frac{1}{2}(dE/E)$, and with $E = \frac{1}{2}m_e(\beta c)^2 = 230$ eV, $dp/p = \frac{1}{2}(0.1/230) = 2.17\text{x}10^{-4}$, so the muons will cool longitudinally to an equilibrium less than the transverse equilibrium. Hence, it is predicted that emittance reduction by a factor of at least 4574 will occur in all three phase space planes, for a total reduction in excess of $9.57\text{x}10^{10}$.

With the physics calculations indicating the feasibility and desirability of the process, the next issue becomes whether or not the needed technology can be built. Phase space manipulation devices such as solenoids, dipoles and RF cavities clearly can be constructed, but the issue of appropriate electron beam construction needs to be proven. Use of square 10 cm by 10 cm cathodes in the example above implies a 1 kA beam current in each such electron system, and the specification of $\beta = 0.03$ leads to a proposed electron beam energy of 230 eV and hence there may be concerns about electron beam instability. By immersing the electron beams in a solenoidal guide field for transport, the magnetic pressure exceeds the beam plasma pressure by about four orders of magnitude – hence stable operation should result unless space charge forces cause difficulty. The key to overcoming space charge problems is to trap low energy ions within the electron beam. Low energy ions will be formed by the electron beam as it passes through residual gas left in the system. The gas atoms are ionized, leaving free ions within the system. As shown in Figure 3, it is possible to use electric fields to trap these ions longitudinally and solenoidal and torroidal magnetic fields to trap these ions transversely. Operation of collectors for electron coolers have proven that extremely high space charge neutralization can be achieved in this way. The important issue with regard to forming 1 kA, 230 eV electron beams is to ensure that forces exist to contain ions for the majority of the beam transport region, and to ensure that metal structures (grids) provide proper fields in the remaining accelerating and decelerating regions. As long as the electron beam self space charge potential is less than the beam energy, the electrons will propagate through any region. Maintaining this condition within the neutralization region requires a high level of neutralization, while maintaining this condition within the accelerating and decelerating regions requires using grids with close proximity to each other and with hole sizes small enough so that the free space charge within the holes and between the grids is acceptable.

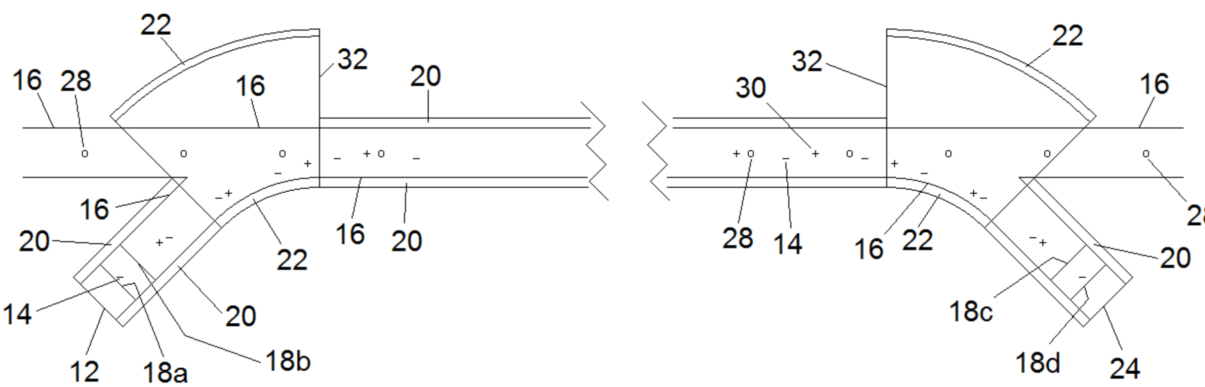

Figure 3. An Electron Cooler for a Muon Beam Filament. Electrons 14 are formed at a standard thermionic cathode 12, accelerated by an electrode 18a and decelerated by an electrode 18b. Vacuum pipes 16 keep background pressures low. The electric field between 18a and 18b provides a force to longitudinally trap neutralizing ions 30 in most of the system. Solenoidal 20 and torroidal 22 windings provide a guiding magnetic field for the electrons, and also provide transverse trapping of the ions 30. Muons 28 enter through the torroid 32, and due to their much larger mass are able to pass through the torroid with only a small bend. Once through the torroid, the muons are overlapped by the electron beam and cooled, and leave through the downstream torroid. The electrons are bent in the downstream torroid and are accelerated between electrodes 18c and 18d, before being decelerated into the electron collector region 24. The electric field between electrodes 18c and 18d forms the downstream end of the longitudinal trap for the neutralizing ions 30.

This letter has outlined one set of parameters for achieving a muon source useful for a muon collider, but several other parameter sets could be considered. Rather than 10 cm by 10 cm, 1 kA electron beams, one could consider 1 cm by 1 cm, 10 A beams, or even smaller ones should electron beam instability prove difficult at the higher current levels. Of course, if smaller coolers are used, more of them will be required and more (and smaller) filamenting magnetic components would be needed. Also, any additional cooling prior to entry to the electron cooler would be very beneficial, due to the cubic dependence of the cooling time on initial beam angular spreads.

Electron cooling is a very powerful technique for increasing the phase space density of particle beams. It has been calculated here that a phase space reduction of almost 10^{11} is predicted by using certain strategies, and that the reduction can occur in several microseconds. Provided the technology can be proven, electron cooling will be an enabling technology for mankind's next steps at understanding the fundamental makeup of our world.

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