

The quantum luminiferous aether

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Abstract: A solid, two-component, quantum luminiferous aether is proposed to exist. Simple postulates are hypothesized, along with some physical laws and assignments. Derivations then lead to the equations of electrodynamics (Maxwell's equations and the Lorentz force equation), Newton's law of universal gravitation, and to two field-masses. The theory is shown to successfully meet the classic tests of general relativity: calculations for the advance of the perihelia, the Shapiro effect, and the gravitational redshift agree with experiment, and the experimental result concerning the bending of light in gravitational fields is also understood. Additionally, gravitational waves are understood, and the first of the field-masses allows for an understanding of what is presently known as dark matter. A new approach to analyzing dense objects such as white dwarfs and neutron stars is discussed, and since the theory has no singularity, a replacement for black holes is suggested. Replacing relativity with an absolute, realist, and physical model returns us to a flat Euclidean space and a separate time. Absolute simultaneity enables understanding of quantum mechanics. The underlying philosophical grounding is discussed. © 2024 *Physics Essays Publication*. [<http://dx.doi.org/10.4006/0836-1398-37.1.9>]

Résumé: Il est proposé qu'il existe un éther lumineux quantique solide à deux composants. Des postulats simples sont émis, ainsi que certaines lois et attributions physiques. Les dérivations conduisent ensuite aux équations de l'électrodynamique (équations de Maxwell et équation de force de Lorentz), à la loi de la gravitation universelle de Newton et à deux champs de masse. Il est démontré que la théorie répond avec succès aux tests classiques de la Relativité Générale: les calculs pour l'avancée des périhélie, l'effet Shapiro et le décalage vers le rouge gravitationnel sont en accord avec l'expérience, et le résultat expérimental concernant la courbure de la lumière dans les champs gravitationnels est également compris. De plus, les ondes gravitationnelles sont comprises et la première des masses de champ permet de comprendre ce que l'on appelle actuellement la matière noire. Une nouvelle approche pour analyser les objets denses tels que les naines blanches et les étoiles à neutrons est discutée, et comme la théorie n'a pas de singularité, un remplacement des trous noirs est suggéré. Remplacer la relativité par un modèle absolu, réaliste et physique nous ramène à un espace euclidien plat et à un temps séparé. La simultanéité absolue permet de comprendre la mécanique quantique. Le fondement philosophique sous-jacent est discuté

Key words: Aether; Electrodynamics; Maxwell's Equations; Lorentz Force Equation; Gravity; Newton's Law of Universal Gravitation; General Relativity; Dark Matter.

FORWARD

This paper is related to a rigorous, quite lengthy, and extremely detailed version available online.¹ In the present work, we will use the equation numbers from Ref. 1 to facilitate cross referencing to Ref. 1 should more details be desired.

PART A. INTRODUCTION

A.1. Present problems in physics

There are presently some rather fundamental problems in physics. While not a problem "per se", the relativity of Einstein^{2,3} is a point-like theory in a curved four-dimensional space-time continuum. Relativity has enjoyed great success in mathematically modeling the effects of

gravity, but its point-like nature leads to infinities. Since particles have finite mass and charge, assuming a volume of zero results in infinite densities. Additionally, there are other problems. Relativity describes black holes as infinite singularities; dark matter particles have not been found;^{4,5} the cosmological constant is orders of magnitude removed from expectations⁶ and quantum mechanics is incompatible with relativity.⁷⁻⁹

The known problems in physics are tolerated because both relativity and quantum mechanics are highly successful. Special relativity enables a derivation of the Lorentz transformation. Both the Lorentz force law¹⁰ and Maxwell's equations¹¹ can be put into a form consistent with relativity, and this explains electrodynamics. General relativity explains gravity. Meanwhile, all experiments done to date are in agreement with quantum mechanics.

Yet despite the great success of relativity and quantum mechanics, the known problems leave us unsatisfied.

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Einstein was always troubled by quantum mechanics as exemplified by his famous quip that God does not play dice. And Einstein, Podolsky, and Rosen⁷ (EPR) wrote a paper showing that relativity and quantum mechanics are incompatible. Bell⁸ extended the work of EPR, and Aspect *et al.*⁹ did experimental tests that agreed with quantum mechanics. Given the fundamental confrontation between quantum mechanics and relativity, we might wish to consider the possibility that relativity should be set aside.

A.2. A new starting philosophy

To look for a new solution, we should start by questioning our most basic foundations, and this involves philosophy. Descartes¹² has shown us that philosophy naturally ends with the conclusion that we cannot really know anything for certain other than that we ourselves exist. And that of course would be a dead end for physics; for physics we must start somewhere. The positivism rooted in Hume,¹³ developed by Mach¹⁴ and embraced by Einstein leads us to put our trust only in experimental observations (although it is important to note that Hume also realized that observations cannot be completely trusted either). In essence, the positivist approach goes down the philosophical path to stop at the trusting of observations, without going all the way to the Descartes realization that we can only trust that we exist. Here, we will reject the positivist philosophy and instead choose a different starting point. We will agree with Descartes that nothing in physics can be trusted at all, and then build up from that minimalist certainty by simply asserting a fundamental axiom of our proposed physical philosophy. We will not try to philosophically prove this axiom to be correct; we will merely assert it and see where it leads. The axiom we propose is that a reality exists that cannot be completely observed. Or more concisely, we assert our Fundamental Axiom:

A Partially observable reality exists.

Our fundamental axiom differs from positivism in that we are asserting the existence of an objective reality, whether we can fully observe that reality or not. We elevate that asserted reality to a primacy above observations. Indeed, since reality is asserted to be only partially observable, observations only inform us about a subset of reality and therefore observations are secondary to reality under our assertion.

With our philosophical foundation established, our aims in physics are then fourfold. First, we must physically model our asserted underlying reality; second, we must logically derive mathematical equations to represent that physical model; third, we must assert where the boundary is between what we can observe and what we cannot observe; and fourth, we must measure what is possible to observe and test our equations against those observations. Our physical model will be built with additional assertions and assignments as we proceed. Agreement of our model with observations will be considered to be affirmative evidence of our model. Disagreement between our model and observations will mean we should correct our model and go through the process again. Our physical philosophy is extremely concise as

compared to that of most philosophers, as we accept the simplicity of the Descartes realization that nothing can be proven to be true, and we then simply propose a limited number of axioms and require no proof of them other than that they lead to equations that are in agreement with experimental observations.

Heisenberg's¹⁵ uncertainty principle at the heart of quantum mechanics can be used to illustrate the difference between positivism and the philosophy following from our fundamental axiom, and the Heisenberg uncertainty principle can also define a boundary between what is and is not observable. Heisenberg states that one cannot simultaneously measure both the momentum and the position of any entity down to an arbitrary small accuracy. If we try to measure the position very accurately, it leads to the momentum being known only vaguely, and vice versa. This places limits on our ability to observe any entity. It becomes the limiting boundary of what we can observe. Under a positivist philosophy, the Heisenberg expression $\Delta p \Delta x \geq \hbar/2$ enforces a limit on any further assertions concerning an underlying reality. Any subquantum analysis or theorizing is to be rejected since it is unverifiable through observations. Under our fundamental axiom, we accept that we cannot simultaneously measure both momentum and size at a subquantum level, but we are free to assert that a subquantum reality nonetheless exists, and that we may analyze it.

Starting from our assertion that a partially observable physical reality exists, we will next choose to simply set relativity aside and return to the idea of an absolute simultaneity. From there we will go on to develop a physical model of our asserted reality. This approach leaves us in a similar position to where things stood prior to 1905, when an absolute theory was assumed. Space was assumed to be flat and Euclidean, and time was the parameter that orders events. Maxwell had already developed his famous equations. Lorentz and others had developed the Lorentz transformation¹⁶ and the Lorentz force equation.¹⁰ The Michelson Morley¹⁷ result was explained by a physical length contraction. All of this was done before relativity, and all of it assumed an underlying aether and absolute simultaneity. The Lorentz ether theory is fully equivalent mathematically to special relativity; all that is lacking is a replacement for general relativity as well as physical models for Maxwell's equations and the Lorentz force equation.

PART B. THE HYPOTHESIS AND SETTING UP THE ANALYSIS

B.1. The hypothesis

Our hypothesis begins by observing that light is a transversely polarized wave. It is known from normal matter that transversely polarized waves are possible in solids under tension. Hence, we propose that the aether is a solid under tension. Since matter is normally quantized, we propose that the aether is quantized. We also propose that there are two types of aether, one we call positive aether and the other we call negative aether. While the aether is usually in a solid form, we propose that sufficient energy might lead to some of the aether being freed from the solid bonds. We will often refer

to the solid aether as attached-aether, and the free pieces as detached-aether. (Later we will identify detached-aether as electric charge.) We propose that different types of aether may be able to flow through other types of aether. Locally, such as within our galaxy, the aether can be thought of as a gigantic solid block. However, aether at great distances away from us may be moving with respect to us. You can think of the aether as being somewhat similar to a glacier, where distant parts may move with respect to one another, but where an analysis of a small local portion will lead to the conclusion that it is a stationary solid block.

B.2. Notation for aetherial displacements

For our analysis, we define the vector fields \mathbf{P} , \mathbf{N} , \mathbf{P}_G , and \mathbf{N}_G . In the absence of sources or waves, the positive aether will be in a nominal state where it is homogeneous and isotropic and be at rest. Each individual small cube of positive-aether will have some position in that nominal state. When waves, sources, or sinks are present, the aether may move away from its nominal position. \mathbf{P} is the displacement vector of the positive-attached-aether from its nominal position, while \mathbf{N} is the displacement vector for the negative-attached-aether. \mathbf{P} and \mathbf{N} refer to displacements caused by electricity and magnetism effects, while \mathbf{P}_G and \mathbf{N}_G refer to displacements caused by gravitational effects.

B.3. Starting assumptions

We will assume some standard starting assumptions based on empirical work done by physicists over time. We will assume Newton's law $F = dp/dt$ is correct, where p is $\beta\gamma mc$, β is v over c , and $\gamma = [1 - \beta^2]^{-1/2}$. Here, v is the velocity of some arbitrary body, and c is the speed of light. The aether is also assumed to be fermionic, and hence it obeys the Pauli exclusion rule.¹⁸ We assume Schrödinger's equation¹⁹ and $E = mc^2$ are valid. For the vast majority of this work, we will analyze things from a frame of reference at rest with respect to the aether, and a physical length contraction and time dilation of moving bodies is assumed. Finally, we will assume that any distortions of a quantum of aether are small with respect to the size of the quantum.

B.4. The five postulates

Five foundational postulates are asserted as part of our hypothesis:

The Density Postulate. In any volume, the density of the positive-aether equals the density of the negative-aether minus an amount proportional to the extrinsic-energy within the volume.

The Tension Postulate. In the absence of external effects, the tension within a quantum of attached-aether is proportional to the separation of any two parallel faces of its surrounding volume.

The Flow Postulate. When aether of one type flows relative to aether of another type, a flow force is generated that is proportional to the flow, and the force is aligned with the flow.

The Aetherial Displacement-Work Postulate. When attached-aether is displaced, work is done on the aether by

the force fields that is proportional to the force, proportional to the distance of displacement, and proportional to the amount of aether displaced.

The Extrinsic-Energy Force-Reduction Postulate. The presence of extrinsic-energy (mass) decreases the positive (negative) attached-aether tension and the negative (positive) attached-aether quantum-force by an amount proportional to the amount of extrinsic-energy present with a constant of proportionality K_{G1} (K_{G2}).

B.5. The foundational equations

With no extrinsic-energy present, the density postulate is expressed mathematically as

$$\rho_P = \rho_{PA} + \rho_{PD} = \rho_{NA} + \rho_{ND} = \rho_N. \quad (1)$$

Here, $\rho_P(\rho_N)$ is the positive (negative) aether density, $\rho_{PA}(\rho_{NA})$ is the positive (negative) attached-aether density, and $\rho_{PD}(\rho_{ND})$ is the positive (negative) detached-aether density. The tension postulate is expressed mathematically as

$$F_{T\xi} = K_{T\xi 0} \xi_Q. \quad (2)$$

We can integrate the tension force of Eq. (2) to obtain the tension energy

$$E_{T\xi} = (1/2)K_{T\xi 0} \xi_Q^2. \quad (3)$$

In Eqs. (2) and (3), ξ_Q is the distance we stretch the aether, and $K_{T\xi 0}$ is the nominal tension parameter for the tension force.

Next, we will look at the quantum effects. Figure 1 shows forces within an aetherial quantum. Recall that we've assumed that the aether is fermionic. As a result of this, we can analyze the situation for an individual fermion. Each fermion will be a single quantum, surrounded by other quanta. Those other quanta will lead to an exclusion region via the assumed Pauli exclusion rule, and this situation results in each fermion being in a three-dimensional square well potential. Within this cubic well, quantum-pressure will push out, and the tension force will pull in. The equilibrium state will exist when these forces balance each other. Using the assumed Schrödinger equation, the energy of a state within an infinite square well is

$$E_{Q\xi} = K_{Q\xi 0} / \xi_Q^2. \quad (4)$$

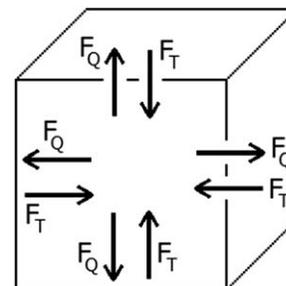


FIG. 1. Forces in an aetherial quantum cube (see Section F.3 for further discussion).

We can now differentiate the energy with respect to distance to get the force magnitude at the cube walls

$$F_{Q\xi} = |dE_{Q\xi}/dX_Q| = 2K_{Q\xi 0}/\xi_Q^3. \quad (5)$$

In Eqs. (4) and (5), $K_{Q\xi 0}$ is the nominal quantum-pressure force parameter. The nominal cube size and aetherial density can be derived (see Ref. 1) as

$$\xi_0 = (2K_{Q\xi 0}/K_{T\xi 0})^{1/4}, \quad (7)$$

$$\begin{aligned} \rho_0 &= Q_\xi/\xi_0^3 = Q_\xi/(2K_{Q\xi 0}/K_{T\xi 0})^{3/4} \\ &= Q_\xi(K_{T\xi 0}/2K_{Q\xi 0})^{3/4}. \end{aligned} \quad (9)$$

Note that in Eq. (9), Q_ξ is the amount of aether contained in a single quantum-cube.

As mentioned in Section A.2, it is possible to analyze the physics of subquantum regions once we adopt our fundamental axiom of a partially observable reality. We will define an analysis-cube as having edge sizes of

$$X_Q = \xi_Q/n. \quad (10)$$

We then form the tension and quantum energies of those cubes

$$E_T = E_{T\xi}/n^3 = (1/2)K_{T\xi 0}\xi_Q^2/n^3 = (1/2)K_{T0}X_Q^2, \quad (11)$$

$$E_Q = E_{Q\xi}/n^3 = K_{Q\xi 0}/\xi_Q^2 n^3 = K_{Q0}/X_Q^2. \quad (12)$$

In Eqs. (11) and (12), we define the new tension and quantum parameters

$$K_{T0} = K_{T\xi 0}/n, \quad (13)$$

$$K_{Q0} = K_{Q\xi 0}/n^5. \quad (14)$$

We then form the forces on the analytic-cube faces by differentiating the energy with respect to distance

$$F_{T0} = |dE_T/dX_Q| = K_{T0}X_Q, \quad (15)$$

$$F_{Q0} = |dE_Q/dX_Q| = 2K_{Q0}/X_Q^3. \quad (16)$$

The advantage of the subquantum analysis is that we can take limits as our analysis-cube shrinks to zero, and this will allow us to drop terms that vanish when we do so.

In Ref. 1, we find the nominal size X_0 of an analysis-cube by evaluating where the total energy $E = E_T + E_Q$ is minimized. We then obtain the following useful relations:

$$X_0^4 = 2K_{Q0}/K_{T0}, \quad (20)$$

$$K_{T0}X_0 = 2K_{Q0}/X_0^3, \quad (21)$$

$$K_{T0}X_0^2 = 2K_{Q0}/X_0^2, \quad (22)$$

$$X_0 = (2K_{Q0}/K_{T0})^{1/4}. \quad (23)$$

Starting from the *Aetherial Displacement-Work Postulate*, Ref. 1 obtains

$$W_{TD} = (X_0/\xi_0)K_c \int K_T X_0 dx, \quad (32)$$

$$W_{QD} = (X_0/\xi_0)K_c \int 2K_Q/X_0^3 dx. \quad (33)$$

In Eqs. (32) and (33), W_{TD} (W_{QD}) is the work done by displacement through the tension (quantum) field, and K_c is the arbitrary proportionality parameter mentioned in the postulate.

PART C. ELECTROMAGNETISM

C.1. Poisson's equation

Recall Eq. (1), the mathematical expression for our density postulate equating the positive and negative aetherial densities: $\rho_P = \rho_{PA} + \rho_{PD} = \rho_{NA} + \rho_{ND} = \rho_N$. If we insert some detached-positive-aether into a region of attached-aether, Eq. (1) states that the total positive-aether density will equal the total negative-aether density, but notice that this could be achieved by either increasing the negative-attached-aether density or decreasing the positive-attached-aether density. Energy considerations can be shown to lead to the positive-attached-aether density being reduced by one half of the inserted positive-detached-aether density, while the negative-attached-aether density is increased by one half of the inserted positive-detached-aether density (see Ref. 1). A similar analysis done for insertion of negative-detached-aether results in

$$\rho_{PA} = \rho_0 - \rho_{PD}/2 + \rho_{ND}/2, \quad (41)$$

$$\rho_{NA} = \rho_0 - \rho_{ND}/2 + \rho_{PD}/2. \quad (42)$$

Figure 2 presents a diagram of what happens when positive-detached-aether is injected into positive-attached-aether. We see that this injection results in an expansion of the positive-attached-aether cube. Prior to the injection of the detached-aether, the amount of positive-attached-aether is the nominal density ρ_0 multiplied by the volume of the cube, $\Delta x \Delta y \Delta z$. When we inject positive-detached-aether into that cube, the amount of positive-attached-aether will remain the same, but the volume of the cube will expand, with Δx becoming larger by δx and similar increases in Δy and Δz . The positive-attached-aether density is then the original amount of aether divided by the new larger volume, and this new density is

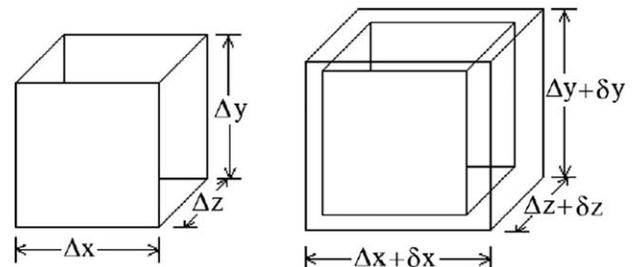


FIG. 2. Analysis-cube of undisturbed aether (left) and one with injected detached-aether (right).

$$\begin{aligned}
 \rho_{\text{PA}} &= \rho_0(\Delta x \Delta y \Delta z) / [(\Delta x + \delta x)(\Delta y + \delta y)(\Delta z + \delta z)] \\
 &\approx \rho_0(\Delta x \Delta y \Delta z) / (\Delta x \Delta y \Delta z + \delta x \Delta y \Delta z \\
 &\quad + \delta y \Delta x \Delta z + \delta z \Delta x \Delta y) \\
 &= \rho_0 / (1 + \delta x / \Delta x + \delta y / \Delta y + \delta z / \Delta z) \\
 &\approx \rho_0(1 - \delta x / \Delta x - \delta y / \Delta y - \delta z / \Delta z).
 \end{aligned} \tag{45}$$

We next observe that the expansion of the cube is related to the difference in the displacement vector \mathbf{P} between its value at the cube center and its value at the edge face of the cube, $\delta x/2 = P_X(x + \Delta x/2, y, z, t) - P_X(x, y, z, t)$. We then divide by $\Delta x/2$ to arrive at $\delta x/\Delta x = [P_X(x + \Delta x/2, y, z, t) - P_X(x, y, z, t)]/\Delta x/2 = \partial P_X/\partial x$, where the last equality is in the limit when we shrink our analysis-cube to zero size. Repeating the derivation for y and z will lead to similar expressions. Hence, Eq. (45) can be re-expressed as $\rho_{\text{PA}} = \rho_0(1 - \delta x/\Delta x - \delta y/\Delta y - \delta z/\Delta z) = \rho_0(1 - \partial P_X/\partial x - \partial P_Y/\partial y - \partial P_Z/\partial z)$, or,

$$\rho_{\text{PA}} = \rho_0(1 - \nabla \cdot \mathbf{P}). \tag{46}$$

We now rearrange the terms of Eq. (46) and then substitute in the value of ρ_{PA} from Eq. (41)

$$\nabla \cdot \mathbf{P} = (\rho_0 - \rho_{\text{PA}})/\rho_0 = (\rho_{\text{PD}} - \rho_{\text{ND}})/2\rho_0. \tag{47}$$

Next we observe from Fig. 2 that \mathbf{P} is purely longitudinal. That is, detached-aether pushes outward on the cube walls but it does not cause any rotation. A purely longitudinal vector field can be formed from the gradient of a scalar field. The scalar field is named Ψ_{P}

$$\mathbf{P}_{\text{L}} = \nabla \Psi_{\text{P}}. \tag{48}$$

This allows us to obtain

$$\nabla \cdot \mathbf{P} = \nabla \cdot \mathbf{P}_{\text{L}} = \nabla \cdot \nabla \Psi_{\text{P}} = \nabla^2 \Psi_{\text{P}}. \tag{49}$$

Next, combine Eqs. (47) and (49) to yield

$$\nabla^2 \Psi_{\text{P}} = \nabla \cdot \mathbf{P}_{\text{L}} = \nabla \cdot \mathbf{P} = (\rho_{\text{PD}} - \rho_{\text{ND}})/2\rho_0. \tag{50}$$

A similar derivation can be applied to the negative-aether to arrive at

$$\nabla^2 \Psi_{\text{N}} = \nabla \cdot \mathbf{N}_{\text{L}} = \nabla \cdot \mathbf{N} = (\rho_{\text{ND}} - \rho_{\text{PD}})/2\rho_0. \tag{52}$$

Subtract Eq. (52) from Eq. (50)

$$\nabla^2(\Psi_{\text{P}} - \Psi_{\text{N}}) = (\rho_{\text{PD}} - \rho_{\text{ND}})/\rho_0. \tag{54}$$

Now define ϕ by $\phi = -(\Psi_{\text{P}} - \Psi_{\text{N}})\rho_0/\epsilon_0$, where ϵ_0 is the permittivity of free space and define $\rho_{\text{D}} = (\rho_{\text{PD}} - \rho_{\text{ND}})$. We have $(\Psi_{\text{P}} - \Psi_{\text{N}}) = -\epsilon_0\phi/\rho_0$ and hence $\nabla^2(\Psi_{\text{P}} - \Psi_{\text{N}}) = -\epsilon_0\nabla^2\phi/\rho_0 = (\rho_{\text{PD}} - \rho_{\text{ND}})/\rho_0 = \rho_{\text{D}}/\rho_0$. Hence, we see that Eq. (54) is Poisson's equation

$$\nabla^2\phi = -\rho_{\text{D}}/\epsilon_0. \tag{55}$$

With this definition for ϕ , we can also derive

$$\mathbf{P}_{\text{L}} - \mathbf{N}_{\text{L}} = \nabla(\Psi_{\text{P}} - \Psi_{\text{N}}) = -\epsilon_0\nabla\phi/\rho_0. \tag{56}$$

We see above that the presence of detached-aether leads to displacement of the negative-attached-aether that is always equal and opposite to the displacement of the positive-attached-aether. Since injection of detached-aether is the only physical cause for the longitudinal displacements \mathbf{P}_{L} and \mathbf{N}_{L} of the attached-aether, we arrive at

$$\mathbf{N}_{\text{L}} = -\mathbf{P}_{\text{L}}. \tag{58}$$

C.2. The delta force

On the walls of a quantum-cube Eq. (15) gives the inward force due to tension as $F_{\text{T0}} = |dE_{\text{T}}/dX_{\text{Q}}| = K_{\text{T0}}X_{\text{Q}}$ and Eq. (16) gives the outward force due to quantum-pressure as $F_{\text{Q0}} = |dE_{\text{Q}}/dX_{\text{Q}}| = 2K_{\text{Q0}}/X_{\text{Q}}^3$. In Section C.1, we have seen that injection of detached-aether changes the size of a positive-attached-aether quantum. Equations (15) and (16) indicate that the increased size will increase the tension and decrease the quantum-pressure. To maintain the larger size X_{Q} , the presence in the cube of positive-detached-aether must therefore result in a detached-aether immersion force \mathbf{F}_{δ} (the delta-force) that acts on the walls of the positive-attached-aether analysis-cube that is equal and opposite to the sum of the longitudinal-tension-force and the quantum-pressure force

$$\mathbf{F}_{\delta} = -\mathbf{F}_{\text{TL}} - \mathbf{F}_{\text{Q}}. \tag{59}$$

The force supplied by the detached-aether is given the name delta force since that force is equal to the difference between the quantum-pressure and tension forces, as that is what is needed to keep the quantum-cube in force equilibrium. Equation (59) above specifies the definition of the delta force. Note that while Eq. (59) appears to form a negative sum, that the direction of the tension is opposite to that of the quantum force, and that is why we say, the delta force is the difference of these two forces. Note also that we use the longitudinal component of the tension force because only the longitudinal component is supplied by the detached-aether. When a source of detached-aether is present, the cubes are displaced and distorted both inside and outside of the source regions. The delta force therefore exists both inside and outside the source regions. The delta force is the force which balances the sum of forces on all cube faces for electromagnetic effects.

C.3. Work done on displaced aether

When a cube of aether moves through the tension and quantum-pressure force fields, work is done as described in Eqs. (32) and (33) above. For the delta force, which equation is used depends upon the form of the delta force for the specific case being evaluated. Reference 1 rigorously evaluates the work done on an analytic-cube as it is displaced against the tension, quantum-pressure, and delta forces. The evaluations are quite tedious, and here we will only present the results. The work done due to tension when a cube moves a distance \mathbf{P} due to a source of detached-aether is

$$\begin{aligned}
 E_{\text{TPPI}} &= K_{\text{T0}}X_0^2 \left[(1/2) + K_{\text{c}}|\mathbf{P}|/\xi_0 + K_{\text{c}}^2(|\mathbf{P}|/\xi_0)^2 \right. \\
 &\quad \left. + 3\delta X/X_0 + 3K_{\text{c}}|\mathbf{P}|\delta X/2X_0\xi_0 \right].
 \end{aligned} \tag{67}$$

In Eq. (67), the subscript TPPI refers to Tension of the Positive-attached-aether due to immersed Positive-detached-aether in the region Inside of a sphere of detached-aether. The cube will also stretch because of the source presence, and δX is the amount it stretches. Equation (67) includes this effect as well. The work done against the quantum-pressure when a cube is moved by \mathbf{P} is

$$E_{\text{QPPI}} = (2K_{\text{Q0}}/X_0^2) \left[(1/2) - K_c |\mathbf{P}| / \xi_0 + K_c^2 |\mathbf{P}|^2 / \xi_0^2 - 3\delta X / X_0 + 3K_c |\mathbf{P}| \delta X / 2X_0 \xi_0 \right]. \quad (69)$$

The work done against the delta force when a cube is moved by \mathbf{P} is

$$E_{\delta\text{PPI}} = 2K_c^2 K_{\text{T0}} (X_0 / \xi_0)^2 |\mathbf{P}|^2 - 3K_{\text{T0}} X_0 K_c |\mathbf{P}| \delta X / \xi_0. \quad (71)$$

The total work is the sum of the work done against the tension, quantum-pressure, and delta forces

$$E_{\text{P}} = K_{\text{T0}} X_0^2 \left[1 + 4K_c^2 P^2 / \xi_0^2 \right]. \quad (72)$$

The total work done via displacement for the negative-aether is

$$E_{\text{N}} = K_{\text{T0}} X_0^2 \left[1 + 4K_c^2 N^2 / \xi_0^2 \right]. \quad (77)$$

C.4. The flow force

Section B.4 specified a flow postulate. Empirically, we now propose a more specific form of the flow postulate for the case of detached-aether flows:

The Electrodynamic Flow Force Law: In regions where there is flowing detached-aether, both the detached-aether and the attached-aether density disturbances caused by the detached-aether will generate a force upon the attached-aether components that is proportional to the relative flow between the flowing detached-aether and the attached-aether; the force will be aligned with the flow, and only the transverse component of the flow leads to a force. (The longitudinal component of the flow does not lead to any force.)

For flowing detached-aether, the force given by *The Electrodynamic Flow Force Law* results in

$$\mathbf{F}_{\text{FP1}} = K_{\text{F1}} \Delta V \rho_{\text{PD}} [\mathbf{U}_{\text{PDT}} - \partial \mathbf{P}_{\text{T}} / \partial t] - K_{\text{F1}} \Delta V \rho_{\text{ND}} [\mathbf{U}_{\text{NDT}} - \partial \mathbf{P}_{\text{T}} / \partial t], \quad (78)$$

$$\mathbf{F}_{\text{FN1}} = K_{\text{F1}} \Delta V \rho_{\text{ND}} [\mathbf{U}_{\text{NDT}} - \partial \mathbf{N}_{\text{T}} / \partial t] - K_{\text{F1}} \Delta V \rho_{\text{PD}} [\mathbf{U}_{\text{PDT}} - \partial \mathbf{N}_{\text{T}} / \partial t]. \quad (79)$$

In Eqs. (78) and (79), K_{F1} is the proportionality constant, ΔV is the volume containing the detached-aether, ρ_{PD} is the positive-detached-aether density, and \mathbf{U}_{PDT} is the transverse velocity of the positive-detached-aether. The negative-aether quantities use a similar nomenclature.

Flowing detached-aether will also cause forces from an “image charge” of flowing attached-aether

$$\mathbf{F}_{\text{FP2}} = -K_{\text{F2}} \Delta V \rho_{\text{PD}} [\mathbf{U}_{\text{PDT}} - \partial \mathbf{P}_{\text{T}} / \partial t] + K_{\text{F2}} \Delta V \rho_{\text{ND}} [\mathbf{U}_{\text{NDT}} - \partial \mathbf{P}_{\text{T}} / \partial t], \quad (80)$$

$$\mathbf{F}_{\text{FN2}} = -K_{\text{F2}} \Delta V \rho_{\text{ND}} [\mathbf{U}_{\text{NDT}} - \partial \mathbf{N}_{\text{T}} / \partial t] + K_{\text{F2}} \Delta V \rho_{\text{PD}} [\mathbf{U}_{\text{PDT}} - \partial \mathbf{N}_{\text{T}} / \partial t]. \quad (81)$$

In Eqs. (80) and (81), K_{F2} is the proportionality constant, which we allow to be different from K_{F1} since the underlying physical cause is different for this flow.

C.5. The transverse tension force

When cubes become transversely displaced from one another, this can lead to a transverse tension force. Figure 3 shows a situation where the center cube is displaced below adjacent cubes. We choose a coordinate system such that the center cube is at a position \mathbf{r} plus $\mathbf{P}(\mathbf{r})$, where \mathbf{r} is the nominal position of the cube for an aether without any sources, sinks or waves. The cube to the right will be located at $\mathbf{r} + \Delta x \mathbf{i} + \mathbf{P}(\mathbf{r} + \Delta x \mathbf{i})$. We can now form a vector \mathbf{D} for the separation of the cubes

$$\begin{aligned} \mathbf{D} &= \mathbf{r} + \Delta x \mathbf{i} + \mathbf{P}(x + \Delta x, y, z, t) - \mathbf{r} - \mathbf{P}(x, y, z, t) \\ &= \Delta x \mathbf{i} + \mathbf{P}(x + \Delta x, y, z, t) - \mathbf{P}(x, y, z, t). \end{aligned} \quad (82)$$

Now form $\mathbf{D}/\Delta x$

$$\begin{aligned} \mathbf{D}/\Delta x &= \mathbf{i} + \partial \mathbf{P} / \partial x \\ &= \mathbf{i} + (\partial P_x / \partial x) \mathbf{i} + (\partial P_y / \partial x) \mathbf{j} + (\partial P_z / \partial x) \mathbf{k}. \end{aligned} \quad (83)$$

Keeping terms to first order in small quantities the magnitude of $\mathbf{D}/\Delta x$ is

$$\begin{aligned} |\mathbf{D}/\Delta x| &= \left[(1 + \partial P_x / \partial x)^2 + (\partial P_y / \partial x)^2 + (\partial P_z / \partial x)^2 \right]^{1/2} \\ &\approx 1 + \partial P_x / \partial x. \end{aligned} \quad (84)$$

Next, form a unit vector \mathbf{d} in the direction of $\mathbf{D}/\Delta x$,

$$\begin{aligned} \mathbf{d} &= (\mathbf{D}/\Delta x) / |\mathbf{D}/\Delta x| \\ &\approx [(1 + \partial P_x / \partial x) \mathbf{i} + (\partial P_y / \partial x) \mathbf{j} \\ &\quad + (\partial P_z / \partial x) \mathbf{k}] / [1 + \partial P_x / \partial x] \\ &\approx \mathbf{i} + (\partial P_y / \partial x) \mathbf{j} + (\partial P_z / \partial x) \mathbf{k}. \end{aligned} \quad (85)$$

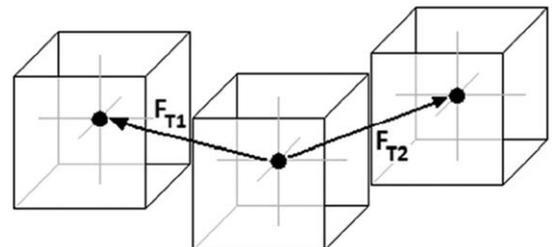


FIG. 3. A central cube displaced downward, leading to tension forces with upward components.

The total force on the central cube of Fig. 3 is

$$\begin{aligned} \mathbf{F}_{\text{TYZ}} = & [F_T(x, y, z, t) + (\partial F_T/\partial x)\Delta x] \\ & [\mathbf{i} + \{\partial P_y(x + \Delta x, y, z, t)/\partial x\}\mathbf{j} \\ & + \{\partial P_z(x + \Delta x, y, z, t)/\partial x\}\mathbf{k}] \\ & - F_T(x, y, z, t)[\mathbf{i} + \{\partial P_y(x, y, z, t)/\partial x\}\mathbf{j} \\ & + \{\partial P_z(x, y, z, t)/\partial x\}\mathbf{k}]. \end{aligned} \quad (88)$$

Equation (88) is simply the force $\mathbf{F}_{\text{T2}} + \mathbf{F}_{\text{T1}}$ from Fig. 3. Note that our arbitrary coordinate system now uses \mathbf{r} as the position of the left face of the central cube and \mathbf{r} plus $\Delta x\mathbf{i}$ for the position of the right face.

Next, we define T_0 as the nominal magnitude of the attached-aetherial-tension per unit area. (T_0 is the tension per unit area in the absence of sources, sinks and waves.) It is assumed that deviations of $F_T(x, y, z, t)$ from $T_0\Delta y\Delta z$ will always be small, or $F_T(x, y, z, t) \approx T_0\Delta y\Delta z$, leaving

$$\begin{aligned} \mathbf{F}_{\text{TYZ}} = & [T_0\Delta y\Delta z + (\partial F_T/\partial x)\Delta x] \\ & [\mathbf{i} + \{\partial P_y(x + \Delta x, y, z, t)/\partial x\}\mathbf{j} \\ & + \{\partial P_z(x + \Delta x, y, z, t)/\partial x\}\mathbf{k}] \\ & - T_0\Delta y\Delta z[\mathbf{i} + \{\partial P_y(x, y, z, t)/\partial x\}\mathbf{j} \\ & + \{\partial P_z(x, y, z, t)/\partial x\}\mathbf{k}]. \end{aligned} \quad (89)$$

It is assumed that $\partial P_y/\partial x$ and $\partial P_z/\partial x$ are small quantities in comparison to 1. Also we note that $\{\partial P_y(x + \Delta x, y, z, t)/\partial x\} - \{\partial P_y(x, y, z, t)/\partial x\} = \Delta x \partial^2 P_y/\partial x^2$ in the limit as $\Delta x \rightarrow 0$ and we have a similar equation for $\partial^2 P_z/\partial x^2$. Keeping only the terms that are lowest order in small quantities

$$\begin{aligned} \mathbf{F}_{\text{TYZ}} = & (\partial F_T/\partial x)\Delta x\mathbf{i} + T_0\Delta x\Delta y\Delta z(\partial^2 P_y/\partial x^2)\mathbf{j} \\ & + T_0\Delta x\Delta y\Delta z(\partial^2 P_z/\partial x^2)\mathbf{k}. \end{aligned} \quad (90)$$

The calculation of the partial derivative of the tension force with respect to x is a bit tedious, and so we do not present it in full here (for full details, see Ref. 1). Here, we just present the result

$$\begin{aligned} (\partial F_T/\partial x)\Delta x = & (\partial F_T/\partial x)X_0 = T_0X_0^3[\partial^2 P_x/\partial x^2] \\ = & \Delta x\Delta y\Delta zT_0(\partial^2 P_x/\partial x^2). \end{aligned} \quad (94)$$

Note again that here we have used $\Delta x = \Delta y = \Delta z = X_0$ and we allow our analysis-cube to shrink to zero. Substituting Eq. (94) into Eq. (90) leaves

$$\begin{aligned} \mathbf{F}_{\text{TYZ}} = & \Delta x\Delta y\Delta z[T_0(\partial^2 P_x/\partial x^2)\mathbf{i} \\ & + T_0(\partial^2 P_y/\partial x^2)\mathbf{j} + T_0(\partial^2 P_z/\partial x^2)\mathbf{k}]. \end{aligned} \quad (95)$$

So far we have just analyzed the forces on the yz cube faces. Applying a similar analysis to the other four cube faces, we find the total transverse tension force on a positive-attached-aether analytic-cube

$$\mathbf{F}_{\text{TPT}} = \Delta x\Delta y\Delta zT_0\nabla^2\mathbf{P}. \quad (96)$$

C.6. Maxwell's equations

We now define $K_{F3} = K_{F1} - K_{F2}$ and then sum Eqs. (78) and (80)

$$\begin{aligned} \mathbf{F}_{\text{FP}} = & \mathbf{F}_{\text{FP1}} + \mathbf{F}_{\text{FP2}} \\ = & -K_{F3}\Delta V\rho_{\text{PD}}[\mathbf{U}_{\text{PDT}} - \partial\mathbf{P}_{\text{T}}/\partial t] \\ & + K_{F3}\Delta V\rho_{\text{ND}}[\mathbf{U}_{\text{NDT}} - \partial\mathbf{P}_{\text{T}}/\partial t]. \end{aligned} \quad (99)$$

Equation (99) is the equation for the total flow force on the positive-attached-aether. Reference 1 shows that $\partial\mathbf{P}/\partial t \ll \mathbf{U}_{\text{PD}}$ so we can drop $\partial\mathbf{P}/\partial t$ and expanding the volume of the analysis-cube ΔV as $\Delta x\Delta y\Delta z$

$$\begin{aligned} \mathbf{F}_{\text{FP}} = & -K_{F3}\rho_{\text{PD}}\Delta x\Delta y\Delta z\mathbf{U}_{\text{PDT}} + K_{F3}\rho_{\text{ND}}\Delta x\Delta y\Delta z\mathbf{U}_{\text{NDT}} \\ = & K_{F3}\Delta x\Delta y\Delta z[\rho_{\text{ND}}\mathbf{U}_{\text{NDT}} - \rho_{\text{PD}}\mathbf{U}_{\text{PDT}}]. \end{aligned} \quad (100)$$

The longitudinal portion of the tension force is balanced by the quantum-pressure and delta forces, and only the transverse portion of the tension force will affect the attached-aether equation of motion. Hence, the force on a cube of attached-aether is given by the sum of the forces given in Eqs. (96) and (100). We can now form $F = ma$ on a cube of the attached-aether. The mass is its mass density m_0 times its volume, $m = m_0\Delta x\Delta y\Delta z$, and the acceleration is the second derivative with respect to time of the cube position. The position is given as the constant nominal position plus \mathbf{P} , and since the derivative of a constant is zero, the acceleration is $\partial^2\mathbf{P}/\partial t^2$. Hence,

$$\begin{aligned} \mathbf{F}_{\text{P}} = & m\mathbf{a} = m_0\Delta x\Delta y\Delta z(\partial^2\mathbf{P}/\partial t^2) = \mathbf{F}_{\text{TPT}} + \mathbf{F}_{\text{FP}} \\ = & \Delta x\Delta y\Delta zT_0\nabla^2\mathbf{P} + K_{F3}\Delta x\Delta y\Delta z[\rho_{\text{ND}}\mathbf{U}_{\text{NDT}} - \rho_{\text{PD}}\mathbf{U}_{\text{PDT}}]. \end{aligned} \quad (101)$$

Now define \mathbf{J} and T_0 and cancel $\Delta x\Delta y\Delta z$ from all terms

$$\mathbf{J} = \rho_{\text{PD}}\mathbf{U}_{\text{PD}} - \rho_{\text{ND}}\mathbf{U}_{\text{ND}}, \quad (103)$$

$$m_0(\partial^2\mathbf{P}/\partial t^2) = T_0\nabla^2\mathbf{P} - K_{F3}\mathbf{J}_{\text{T}}, \quad (104)$$

$$T_0 = m_0c^2, \quad (107)$$

$$\nabla^2\mathbf{P} - (1/c^2)(\partial^2\mathbf{P}/\partial t^2) = (K_{F3}/T_0)\mathbf{J}_{\text{T}}. \quad (108)$$

Next we take the perpendicular component of Eq. (108) and use a similar analysis on the negative-attached-aether

$$\nabla^2\mathbf{P}_{\text{T}} - (1/c^2)(\partial^2\mathbf{P}_{\text{T}}/\partial t^2) = (K_{F3}/T_0)\mathbf{J}_{\text{T}}, \quad (110)$$

$$\nabla^2\mathbf{N}_{\text{T}} - (1/c^2)(\partial^2\mathbf{N}_{\text{T}}/\partial t^2) = -(K_{F3}/T_0)\mathbf{J}_{\text{T}}. \quad (111)$$

It can be seen that Eq. (110) for \mathbf{P}_{T} is nearly identical to Eq. (111) for \mathbf{N}_{T} , as they only differ in the sign of the right-hand side and replacement of \mathbf{P}_{T} by \mathbf{N}_{T} , and so we have $\mathbf{N}_{\text{T}} = -\mathbf{P}_{\text{T}}$, and recalling Eq. (58), $\mathbf{N}_{\text{L}} = -\mathbf{P}_{\text{L}}$,

$$\mathbf{N} = -\mathbf{P}. \quad (113)$$

We will now define a vector \mathbf{A} ,

$$\mathbf{P}_{\text{T}} = -\mathbf{N}_{\text{T}} = -K_{F3}\mathbf{A}/\mu_0T_0. \quad (114)$$

Substituting the relevant equation of Eqs. (114) into Eqs. (110) and (111) leaves

$$\nabla^2 \mathbf{A} - (1/c^2)(\partial^2 \mathbf{A}/\partial t^2) = -\mu_0 \mathbf{J}_T. \quad (115)$$

We will choose the conventional definition of \mathbf{J}_L ,

$$\mathbf{J}_L = -(1/4\pi) \nabla \int [(\nabla' \cdot \mathbf{J})/|\mathbf{x} - \mathbf{x}'|] d^3x'. \quad (116)$$

Next note that the solution to Poisson's equation, Eq. (55), $\nabla^2 \phi = -\rho_D/\epsilon_0$, is

$$\phi(x, t) = (1/4\pi\epsilon_0) \int [\rho_D(\mathbf{x}', t)/|\mathbf{x} - \mathbf{x}'|] d^3x'. \quad (117)$$

Now take the gradient of $\phi(x, t)$ and then take its partial time derivative

$$\partial \nabla \phi(x, t)/\partial t = (1/4\pi\epsilon_0) \nabla \int \partial [\rho_D(\mathbf{x}', t)/\partial t / |\mathbf{x} - \mathbf{x}'|] d^3x'. \quad (118)$$

Using the continuity equation, $\partial \rho_D/\partial t = -\nabla \cdot \mathbf{J}$, Eqs. (116) and (118) reveal

$$\epsilon_0 \partial \nabla \phi(x, t)/\partial t = \mathbf{J}_L. \quad (119)$$

At this point, we can therefore add the quantity $-\mu_0 \mathbf{J}_L + \epsilon_0 \mu_0 \partial \nabla \phi/\partial t$ to Eq. (115), since this is adding zero. And with $\epsilon_0 \mu_0 = 1/c^2$, we get

$$\nabla^2 \mathbf{A} - (1/c^2)(\partial^2 \mathbf{A}/\partial t^2) = -\mu_0 \mathbf{J}_T - \mu_0 \mathbf{J}_L + (1/c^2) \partial \nabla \phi/\partial t \quad (120)$$

or

$$\nabla^2 \mathbf{A} - (1/c^2)(\partial^2 \mathbf{A}/\partial t^2) = -\mu_0 \mathbf{J} + (1/c^2) \partial \nabla \phi/\partial t. \quad (121)$$

Furthermore, since Eq. (114) informs us that \mathbf{A} is a transverse vector

$$\nabla \cdot \mathbf{A} = 0. \quad (122)$$

It is also timely to recall Eq. (55)

$$\nabla^2 \phi = -\rho_D/\epsilon_0. \quad (55)$$

Equations (55), (121), and (122) are readily recognized as Maxwell's equations in the Coulomb gauge in terms of potentials.

To get the more familiar Maxwell's equations in terms of fields, we start by defining two arbitrary vectors \mathbf{E} and \mathbf{B} ,

$$\mathbf{E} = -\nabla \phi - \partial \mathbf{A}/\partial t, \quad (123)$$

$$\mathbf{B} = \nabla \times \mathbf{A}. \quad (124)$$

Applying the partial derivative with respect to time to Eq. (123)

$$\partial \mathbf{E}/\partial t = -\partial \nabla \phi/\partial t - \partial^2 \mathbf{A}/\partial t^2. \quad (125)$$

Next, add a term $\nabla(\nabla \cdot \mathbf{A})$ to Eq. (121), which is permissible since by Eq. (122) $\nabla \cdot \mathbf{A} = 0$

$$\nabla^2 \mathbf{A} - (1/c^2)(\partial^2 \mathbf{A}/\partial t^2) = -\mu_0 \mathbf{J} + (1/c^2) \partial \nabla \phi/\partial t + \nabla(\nabla \cdot \mathbf{A}). \quad (126)$$

Now rearrange terms (first equality below) and use Eq. (125) for $\partial \mathbf{E}/\partial t$ (second equality below)

$$\begin{aligned} \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \\ = \mu_0 \mathbf{J} - (1/c^2) \partial \nabla \phi/\partial t - (1/c^2)(\partial^2 \mathbf{A}/\partial t^2) \\ = \mu_0 \mathbf{J} + (1/c^2) \partial \mathbf{E}/\partial t. \end{aligned} \quad (127)$$

Now use $\nabla \times \mathbf{B} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$ to get

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + (1/c^2) \partial \mathbf{E}/\partial t. \quad (128)$$

Taking the divergence of Eq. (123), $\nabla \cdot \mathbf{E} = -\nabla \cdot \nabla \phi - \partial(\nabla \cdot \mathbf{A})/\partial t$. With $\nabla \cdot \mathbf{A} = 0$, and $\nabla \cdot \nabla \phi = \nabla^2 \phi$, and with Eq. (55) $\nabla^2 \phi = -\rho_D/\epsilon_0$ this leaves

$$\nabla \cdot \mathbf{E} = \rho_D/\epsilon_0. \quad (129)$$

Taking the divergence of Eq. (124), along with the identity $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

$$\nabla \cdot \mathbf{B} = 0. \quad (130)$$

Now take the curl of Eq. (123), $\nabla \times \mathbf{E} = -\nabla \times \nabla \phi - \partial(\nabla \times \mathbf{A})/\partial t$ and recalling Eq. (124) $\mathbf{B} = \nabla \times \mathbf{A}$ and using the identity $\nabla \times \nabla \phi = 0$, we get

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t. \quad (131)$$

Equations (128)–(131) are Maxwell's equations in terms of fields.

C.7. The Lorentz force equation

So far we have looked at the forces on the attached-aether. Now we will look at the forces on the detached-aether, taking note that the density of the detached-aether ρ_D in Eqs. (55) and (129) is now recognized as what we call electric charge density.

Equations (72) and (77) enable calculation of the force that will occur on a sphere of detached-aether in the presence of an ambient aetherial displacement $\mathbf{P}_{AL} = -\mathbf{N}_{AL}$. We assert that we can build any arbitrary distribution of detached-aether from small enough uniform spheres, and that the superposition of the resulting fields will allow for the calculation of the general situation.

To analyze what happens to a sphere of detached-aether in the presence of an ambient aetherial displacement, we will first divide the sphere into slices, and then divide the slices into strips as shown in Fig. 4. We will then look at small cubes within those strips. The force on each cube will be evaluated, and the force from all of the cubes will be summed to find the total force on the sphere.

If there is no ambient displacement of the aether, the detached-aether will push the attached-aether out radially. That displacement will be zero at the center of the sphere and increase radially until the edge, as this is a solution of Eq. (55)

$$\mathbf{P}_{\text{sphere}} = (\rho_{\text{PD}}/6\rho_0)\mathbf{r} = (\rho_{\text{PD}}/6\rho_0)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}). \quad (132)$$

If there is now an ambient longitudinal positive-attached-aether displacement \mathbf{P}_{AL} , which without loss of generality can be considered to be in the X direction, $\mathbf{P}_{\text{AL}} = P_{\text{A}}\mathbf{i}$, then the total positive-attached-aether displacement within the sphere becomes $\mathbf{P}_{\text{TOT}} = \mathbf{P}_{\text{AL}} + \mathbf{P}_{\text{sphere}} = [(\rho_{\text{PD}}/6\rho_0)x + P_{\text{A}}]\mathbf{i} + (\rho_{\text{PD}}/6\rho_0)y\mathbf{j} + (\rho_{\text{PD}}/6\rho_0)z\mathbf{k}$. We can now calculate the displacement energy of the sphere from Eq. (72)

$$\begin{aligned} E_{\text{PP}}(x, y, z) &= 4K_c^2 K_{\text{T0}} X_0^2 P^2 / \xi_0^2 = 4K_c^2 K_{\text{T0}} (X_0^2 / \xi_0^2) [P_{\text{TOT}}(x, y, z)]^2 \\ &= 4K_c^2 K_{\text{T0}} (X_0^2 / \xi_0^2) \left[(\rho_{\text{PD}}/6\rho_0)^2 x^2 + 2(\rho_{\text{PD}}/6\rho_0)xP_{\text{A}} \right. \\ &\quad \left. + P_{\text{A}}^2 + (\rho_{\text{PD}}/6\rho_0)^2 y^2 + (\rho_{\text{PD}}/6\rho_0)^2 z^2 \right]. \end{aligned} \quad (133)$$

We then calculate the effect of a virtual displacement δx and that gives us

$$\begin{aligned} E_{\text{PP}}(x + \delta x, y, z) &= 4K_c^2 K_{\text{T0}} (X_0^2 / \xi_0^2) [P_{\text{TOT}}(x + \delta x, y, z)]^2 \\ &= 4K_c^2 K_{\text{T0}} (X_0^2 / \xi_0^2) \left[(\rho_{\text{PD}}/6\rho_0)^2 (x + \delta x)^2 \right. \\ &\quad \left. + 2(\rho_{\text{PD}}/6\rho_0)(x + \delta x)P_{\text{A}} + P_{\text{A}}^2 \right. \\ &\quad \left. + (\rho_{\text{PD}}/6\rho_0)^2 y^2 + (\rho_{\text{PD}}/6\rho_0)^2 z^2 \right]. \end{aligned} \quad (134)$$

Subtracting Eq. (133) from Eq. (134) leaves the energy change resulting from the virtual displacement

$$\begin{aligned} \delta E_{\text{PP}}(x, y, z) &= E_{\text{PP}}(x + \delta x, y, z) - E_{\text{PP}}(x, y, z) \\ &= 4K_c^2 K_{\text{T0}} (X_0^2 / \xi_0^2) \left[2(\rho_{\text{PD}}/6\rho_0)^2 x \delta x \right. \\ &\quad \left. + (\rho_{\text{PD}}/6\rho_0)^2 \delta x^2 + 2(\rho_{\text{PD}}/6\rho_0) \delta x P_{\text{A}} \right]. \end{aligned} \quad (135)$$

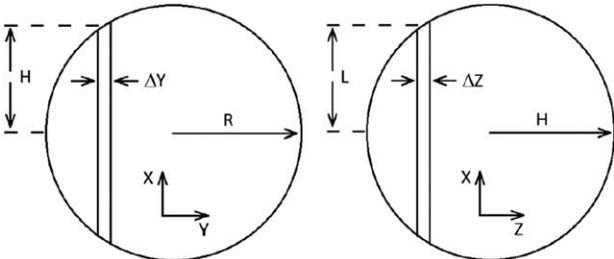


FIG. 4. A sphere of detached-aether, and a slice of width ΔY (left). A slice of the sphere showing a strip of depth ΔZ (right).

In the above expression, we can drop the term that is second order in the small quantity δx , as we will take the limit as $\delta x \rightarrow 0$. We can now evaluate the force on the strip by considering the sum of all volume elements within the strip. We can drop the term $2(\rho_{\text{PD}}/6\rho_0)^2 x \delta x$ because for every value of positive x on our strip there is a value of negative x of equal magnitude. [The term $2(\rho_{\text{PD}}/6\rho_0)^2 x \delta x$ cancels out over the strip because of symmetry.] The surviving term, $8K_c^2 K_{\text{T0}} (X_0^2 / \xi_0^2) (\rho_{\text{PD}}/6\rho_0) \delta x P_{\text{A}}$, is independent of x , y , or z . Recalling that Eq. (135) refers to the change in energy for an analysis-cube within the strip, we can form the relation for the force on the whole strip by summing over all of the analysis-cubes in the strip (Σ_{strip} is the symbol for that sum). Analyzing a strip of length $2L$, width X_0 , and height X_0 , there will be $2L/X_0$ analysis-cubes in that strip. The force in that strip will be

$$\begin{aligned} F_{\text{stripPP}} &= \sum_{\text{strip}} \{ \delta E_{\text{PP}}(x, y, z) / \delta x \} \\ &= (2L/X_0) 8K_c^2 K_{\text{T0}} (X_0^2 / \xi_0^2) (\rho_{\text{PD}}/6\rho_0) \delta x P_{\text{A}} / \delta x \\ &= 8LK_c^2 K_{\text{T0}} X_0 \rho_{\text{PD}} P_{\text{A}} / 3\rho_0 \xi_0^2 \\ &= 8K_c^2 K_{\text{T0}} L X_0^2 \rho_{\text{PD}} P_{\text{A}} / 3\rho_0 \xi_0^2 X_0. \end{aligned} \quad (136)$$

The force on the strip shown in Fig. 4 is proportional to the volume of the strip ($2LX_0^2$) but independent of x , y , and z . The sum of the volume of all of the strips will be the volume of the sphere, V_{sphere} . Hence, we can sum the forces from all such strips to arrive at

$$F_{\text{spherePP}} = 4K_c^2 K_{\text{T0}} V_{\text{sphere}} \rho_{\text{PD}} P_{\text{A}} / 3\rho_0 \xi_0^2 X_0. \quad (137)$$

Including the force from the negative aether will lead to a factor of two increase in the force, and here $P_{\text{A}} = \mathbf{P}_{\text{AL}}$ so the total force is now

$$\mathbf{F}_{\text{sphereP}} = 8K_c^2 K_{\text{T0}} V_{\text{sphere}} \rho_{\text{PD}} \mathbf{P}_{\text{AL}} / 3\rho_0 \xi_0^2 X_0. \quad (139)$$

We now recall Eq. (56) $\mathbf{P}_{\text{L}} - \mathbf{N}_{\text{L}} = -\epsilon_0 \nabla \phi / \rho_0$ and using Eq. (113), $\mathbf{N} = -\mathbf{P}$ we form $\mathbf{P}_{\text{AL}} - \mathbf{N}_{\text{AL}} = 2\mathbf{P}_{\text{AL}} = -\epsilon_0 \nabla \phi / \rho_0$. This allows us to arrive at

$$\mathbf{F}_{\text{sphere}} = -4K_c^2 K_{\text{T0}} V_{\text{sphere}} \rho_{\text{PD}} \epsilon_0 \nabla \phi / 3\xi_0^2 X_0 \rho_0^2. \quad (140)$$

We now set the value of K_{T0}

$$K_{\text{T0}} = 3\rho_0^2 X_0 \xi_0^2 / 4K_c^2 \epsilon_0. \quad (141)$$

And setting $Q = V_{\text{sphere}} \rho_{\text{PD}}$ in Eq. (140), we obtain the Lorentz force on the detached-aether due to ambient displaced aether

$$\mathbf{F}_{\text{LD}} = -Q \nabla \phi. \quad (142)$$

A similar derivation for negative-detached-aether will again result in Eq. (142), provided we make the more general assignment of $Q = V_{\text{sphere}} (\rho_{\text{PD}} - \rho_{\text{ND}})$.

We now consider the flow force on a sphere of detached-aether. The reaction force back on the detached-aether will be the negative of the sum of Eqs. (78) and (79), and this allows us to obtain

$$\begin{aligned}\mathbf{F}_{\text{LF}} &= K_{\text{F1}}\Delta V\rho_{\text{PD}}(\partial\mathbf{P}_{\text{T}}/\partial t - \partial\mathbf{N}_{\text{T}}/\partial t) \\ &\quad - K_{\text{F1}}\Delta V\rho_{\text{ND}}[\partial\mathbf{P}_{\text{T}}/\partial t - \partial\mathbf{N}_{\text{T}}/\partial t] \\ &= K_{\text{F1}}Q(\partial\mathbf{P}_{\text{T}}/\partial t - \partial\mathbf{N}_{\text{T}}/\partial t).\end{aligned}\quad (143)$$

In Eq. (143), Q is defined as $\Delta V(\rho_{\text{PD}} - \rho_{\text{ND}})$ where now $\Delta V = V_{\text{sphere}}$. Recall Eq. (114), $\mathbf{P}_{\text{T}} = -\mathbf{N}_{\text{T}} = -K_{\text{F3}}\mathbf{A}/\mu_0 T_0$. From this, and using Eq. (113), $\mathbf{N} = -\mathbf{P}$, we get $\partial\mathbf{P}_{\text{T}}/\partial t - \partial\mathbf{N}_{\text{T}}/\partial t = -2K_{\text{F3}}(\partial\mathbf{A}/\partial t)/\mu_0 T_0$ so $\mathbf{F}_{\text{LF}} = K_{\text{F1}}Q(\partial\mathbf{P}_{\text{T}}/\partial t - \partial\mathbf{N}_{\text{T}}/\partial t) = -(2K_{\text{F3}}K_{\text{F1}}Q/\mu_0 T_0)\partial\mathbf{A}/\partial t$. Now make the assignment

$$\mu_0 T_0 = 2K_{\text{F3}}K_{\text{F1}}. \quad (144)$$

This allows the Lorentz force on any detached-aether due to flow forces to be expressed as

$$\mathbf{F}_{\text{LF}} = -Q\partial\mathbf{A}/\partial t. \quad (145)$$

The last force on detached-aether to be evaluated involves an interplay between the flow and tension forces, and it involves some complexity. Without loss of generality, we can assume the flow is in the z direction, and this flow will displace the attached-aether in the z direction. An ambient gradient in the attached-aether displacement could be in any of the directions x , y , or z , and we will need to evaluate each of these cases separately. Superposition of the separate solutions will then be done to arrive at the general expression for this force.

We will first consider a gradient of the z component of the aetherial displacement with respect to x . The left side of Fig. 5 shows the situation when there is no detached-aether flowing through the cube. In that ambient condition, a cube to the left of the one shown lies further down, and a cube to the right lies further up. This leads to the force vectors shown, and as can be seen, the forces will be in equilibrium in that case. When there is a flow, as shown on the right side of Fig. 5, the cube that is shown is pushed downward, and this alters the relative position of that cube with respect to its neighbors, altering the force vectors on its right and left faces. Here we have looked at the forces on the y - z faces of the cube.

Figure 6 shows the situation that exists on the x - z faces of the cube when there is an aetherial gradient $\partial P_{\text{AZ}}/\partial x$. When the flow force pushes the cube downward, we again get a restoring upward force from the tension. The equilibrium will occur when the downward force from the flow is balanced by the upward component of the tension forces, and forces shown in Figs. 5 and 6 will all contribute to this equilibrium.

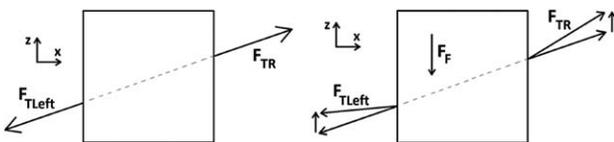


FIG. 5. Tension forces on the y - z faces of an analysis-cube of positive-attached-aether in the presence of an ambient positive-attached-aether displacement with gradient $\partial P_{\text{AZ}}/\partial x$. Left side is when there is no flowing detached-aether, right side when there is flowing detached-aether.

We now define an angle $\theta_{\text{F}} = dP_{\text{F}}/\Delta x$ and evaluate the tension forces shown in Figs. 5 and 6

$$\begin{aligned}\mathbf{F}_{\text{Left}} &\approx T_0\Delta x^2[-\mathbf{i} + (\theta_{\text{F}} - \partial P_{\text{AZ}}/\partial x)\mathbf{k} \\ &\quad + \mathbf{i}(\theta_{\text{F}} - \partial P_{\text{AZ}}/\partial x)^2/2],\end{aligned}\quad (147)$$

$$\mathbf{F}_{\text{Right}} \approx T_0\Delta x^2[\mathbf{i} + (\theta_{\text{F}} + \partial P_{\text{AZ}}/\partial x)\mathbf{k} - \mathbf{i}(\theta_{\text{F}} + \partial P_{\text{AZ}}/\partial x)^2/2], \quad (148)$$

$$\mathbf{F}_{\text{Front}} \approx T_0\Delta x^2[\mathbf{j} + \theta_{\text{F}}\mathbf{k} - \mathbf{j}\theta_{\text{F}}^2/2], \quad (149)$$

$$\mathbf{F}_{\text{Back}} \approx T_0\Delta x^2[-\mathbf{j} + \theta_{\text{F}}\mathbf{k} + \mathbf{j}\theta_{\text{F}}^2/2]. \quad (150)$$

Finally, there is the force from the flow within the cube. Equation (100) above, $\mathbf{F}_{\text{FP}} = -K_{\text{F3}}\rho_{\text{PD}}\Delta V\mathbf{U}_{\text{PDT}} + K_{\text{F3}}\rho_{\text{ND}}\Delta V\mathbf{U}_{\text{NDT}}$, gives the expression for the total flow force on the positive-attached-aether due to flows. Since we are only looking at positive-detached-aether flow, this becomes

$$\mathbf{F}_{\text{FlowPP}} = -K_{\text{F3}}\rho_{\text{PD}}\Delta V\mathbf{U}_{\text{PDT}} = -K_{\text{F3}}Q_{\text{P}}U\mathbf{k}. \quad (151)$$

In the second of Eq. (151), $Q_{\text{P}} = \rho_{\text{PD}}\Delta V$ and $\mathbf{U}_{\text{PDT}} = U\mathbf{k}$. Summing the five forces yields the total force on the cube

$$\mathbf{F}_{\text{LTIPP}} = T_0\Delta x^2[4\theta_{\text{F}}\mathbf{k} - 2\theta_{\text{F}}(\partial P_{\text{AZ}}/\partial x)\mathbf{i}] - K_{\text{F3}}Q_{\text{P}}U\mathbf{k}. \quad (152)$$

The subscript reminds us we are evaluating the Lorentz Tension-force of type 1 for the case of Positive-detached-aether flowing through Positive-attached-aether. In this case, since the cube is free to move in the z direction, equilibrium will be obtained when $4T_0\Delta x^2\theta_{\text{F}} = K_{\text{F3}}Q_{\text{P}}U$ or $\theta_{\text{F}} = K_{\text{F3}}Q_{\text{P}}U/4T_0\Delta x^2$ and hence $\mathbf{F}_{\text{LTIPP}} = -T_0\Delta x^22\theta_{\text{F}}(\partial P_{\text{AZ}}/\partial x)\mathbf{i} = -\mathbf{i}(\partial P_{\text{AZ}}/\partial x)K_{\text{F3}}Q_{\text{P}}U/2$. Now recall $\mathbf{P}_{\text{T}} = -K_{\text{F3}}\mathbf{A}/\mu_0 T_0$ from Eq. (114) to obtain

$$\mathbf{F}_{\text{LTIPP}} = \mathbf{i}(\partial A_{\text{AZ}}/\partial x)K_{\text{F3}}^2Q_{\text{P}}U/2\mu_0 T_0. \quad (153)$$

The positive-detached-aether moves through both types of attached-aether. When we evaluate positive-detached-aether flowing through negative-attached-aether the relevant portions of Eqs. (79) and (81) gives us a flow force of $\mathbf{F}_{\text{FlowPN}} = K_{\text{F3}}\rho_{\text{PD}}\Delta V\mathbf{U}_{\text{PDT}}$, which is the negative of the flow force given by Eq. (151). In this case, we use $\mathbf{N}_{\text{T}} = K_{\text{F3}}\mathbf{A}/\mu_0 T_0$ from Eq. (114), which is the negative of what was used to get to Eq. (153). Since there are two sign reversals in the derivation, we obtain

$$\mathbf{F}_{\text{LTIPN}} = \mathbf{i}(\partial A_{\text{AZ}}/\partial x)K_{\text{F3}}^2Q_{\text{P}}U/2\mu_0 T_0. \quad (154)$$

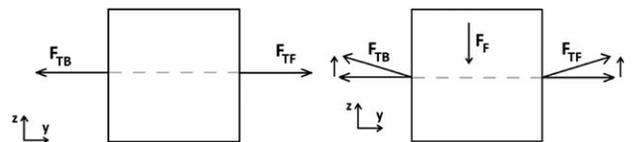


FIG. 6. Tension forces on x - z faces of an analysis-cube of positive-attached-aether in the presence of an ambient positive-attached-aether displacement with gradient $\partial P_{\text{AZ}}/\partial x$. Left side is when there is no flowing detached-aether, right side when there is flowing detached-aether.

Equations (153) and (154) are forces on the attached-aether due to flows of positive-detached-aether. The force on the attached-aether can be cancelled if the force from Eqs. (153) and (154) is transferred to the detached-aether. Hence, the total Lorentz Tension force of type 1 on the positive-detached-aether, \mathbf{F}_{LTIP} , is the sum of Eqs. (153) and (154)

$$\mathbf{F}_{\text{LTIP}} = \mathbf{i}(\partial A_{\text{AZ}}/\partial x)K_{\text{F3}}^2 Q_{\text{P}}U/\mu_0 T_0. \quad (155)$$

For the total Lorentz Tension force of type 1 on negative-detached-aether, Eqs. (78)–(81) inform us that there is simply a change in sign in the direction of the flow force. [Recall that we drop the terms $\partial \mathbf{P}_{\text{T}}/\partial t$ and $\partial \mathbf{N}_{\text{T}}/\partial t$ to get to Eq. (100).] Therefore, the derivation involves replacing Q_{P} by $-Q_{\text{N}}$ which leaves

$$\mathbf{F}_{\text{LTIN}} = -\mathbf{i}(\partial A_{\text{AZ}}/\partial x)K_{\text{F3}}^2 Q_{\text{N}}U/\mu_0 T_0. \quad (156)$$

Defining $Q = Q_{\text{P}} - Q_{\text{N}}$ and combining Eqs. (155) and (156) leaves

$$\mathbf{F}_{\text{LTI}} = \mathbf{i}(\partial A_{\text{AZ}}/\partial x)K_{\text{F3}}^2 QU/\mu_0 T_0. \quad (157)$$

Now, assigning our constant K_{F3} through $K_{\text{F3}}^2 = \mu_0 T_0$ allows us to arrive at

$$\mathbf{F}_{\text{LTI}} = \mathbf{i}(\partial A_{\text{AZ}}/\partial x)QU. \quad (159)$$

A similar treatment to the above can be applied if the ambient, parallel, attached-aetherial displacements are instead assumed to vary in the y direction. This leads to

$$\mathbf{F}_{\text{LT2}} = \mathbf{j}(\partial A_{\text{AZ}}/\partial y)QU. \quad (160)$$

The other possibility for forces arising from detached-aether flowing in the z direction is shown in Fig. 7, where we see the effect of a gradient of the ambient P_{X} with respect to z . In this case, the ambient condition is that of a plane of cubes above the cube shown being displaced to the right,

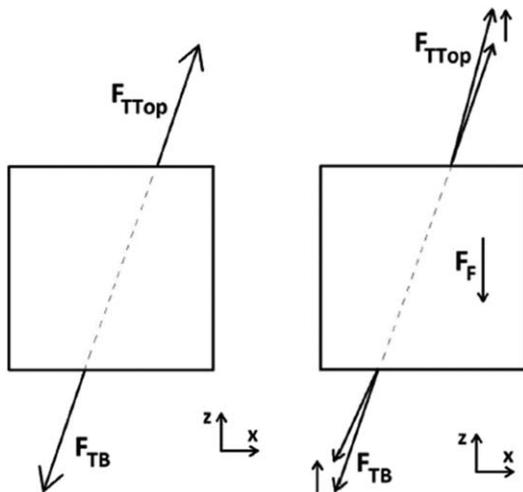


FIG. 7. Tension-forces on the x - y faces of an analysis-cube of attached-aether in the presence of an ambient aetherial gradient $\partial P_{\text{AX}}/\partial z$. Left side shows the case where there is no flowing detached-aether, right side shows the case where there is flowing detached-aether.

while a plane of cubes below the one shown is displaced to the left. In the left figure we can see that without any flowing detached-aether, there is a simple force balance. In the right figure we see the effect when flowing detached-aether forces the cube downward. The downward displacement of the cube will alter the top and bottom tension forces resulting in an upward net tension force that partially balances the force from the downward flow force. (Forces on the remaining cube faces complete the force balance relationship in the z direction.)

The evaluation of the coupled tension and flow forces in the presence of a gradient of P_{AX} with respect to z for the case of detached-aether moving in the z direction is rather tedious and for that reason we will refer to Ref. 1 for all of the details. Here, we present the result of the analysis

$$\mathbf{F}_{\text{LT3}} = -\mathbf{i}(\partial A_{\text{AX}}/\partial z)QU, \quad (169)$$

$$\mathbf{F}_{\text{LT4}} = -\mathbf{j}(\partial A_{\text{AY}}/\partial z)QU. \quad (170)$$

Summing the forces of Eqs. (159), (160), (169), and (170), we obtain

$$\mathbf{F}_{\text{LT5}} = \mathbf{i}(\partial A_{\text{AZ}}/\partial x)QU_{\text{Z}} + \mathbf{j}(\partial A_{\text{AZ}}/\partial y)QU_{\text{Z}} - \mathbf{i}(\partial A_{\text{AX}}/\partial z)QU_{\text{Z}} - \mathbf{j}(\partial A_{\text{AY}}/\partial z)QU_{\text{Z}}. \quad (171)$$

In general, the velocity may not be in the z direction. With a velocity in the x direction, $U = U_{\text{X}}$, we just make the substitutions z to x , x to y , y to z , \mathbf{i} to \mathbf{j} , \mathbf{j} to \mathbf{k} , and \mathbf{k} to \mathbf{i} , in the analysis. Then we do the same substitutions once more to obtain the expression when $U = U_{\text{Y}}$,

$$\mathbf{F}_{\text{LT6}} = \mathbf{j}(\partial A_{\text{AX}}/\partial y)QU_{\text{X}} + \mathbf{k}(\partial A_{\text{AX}}/\partial z)QU_{\text{X}} - \mathbf{j}(\partial A_{\text{AY}}/\partial x)QU_{\text{X}} - \mathbf{k}(\partial A_{\text{AZ}}/\partial x)QU_{\text{X}}, \quad (172)$$

$$\mathbf{F}_{\text{LT7}} = \mathbf{k}(\partial A_{\text{AY}}/\partial z)QU_{\text{Y}} + \mathbf{i}(\partial A_{\text{AY}}/\partial x)QU_{\text{Y}} - \mathbf{k}(\partial A_{\text{AZ}}/\partial y)QU_{\text{Y}} - \mathbf{i}(\partial A_{\text{AX}}/\partial y)QU_{\text{Y}}. \quad (173)$$

The total Lorentz force on moving detached-aether due to the tension/flow effect is

$$\begin{aligned} F_{\text{LT}} &= F_{\text{LT5}} + F_{\text{LT6}} + F_{\text{LT7}} \\ &+ \mathbf{i}(\partial A_{\text{AZ}}/\partial x)QU_{\text{Z}} + \mathbf{j}(\partial A_{\text{AZ}}/\partial y)QU_{\text{Z}} \\ &- \mathbf{i}(\partial A_{\text{AX}}/\partial z)QU_{\text{Z}} - \mathbf{j}(\partial A_{\text{AY}}/\partial z)QU_{\text{Z}} \\ &+ \mathbf{j}(\partial A_{\text{AX}}/\partial y)QU_{\text{X}} + \mathbf{k}(\partial A_{\text{AX}}/\partial z)QU_{\text{X}} \\ &- \mathbf{j}(\partial A_{\text{AY}}/\partial x)QU_{\text{X}} - \mathbf{k}(\partial A_{\text{AZ}}/\partial x)QU_{\text{X}} \\ &+ \mathbf{k}(\partial A_{\text{AY}}/\partial z)QU_{\text{Y}} + \mathbf{i}(\partial A_{\text{AY}}/\partial x)QU_{\text{Y}} \\ &- \mathbf{k}(\partial A_{\text{AZ}}/\partial y)QU_{\text{Y}} - \mathbf{i}(\partial A_{\text{AX}}/\partial y)QU_{\text{Y}}. \end{aligned} \quad (174)$$

The total force on a volume of detached-aether, Q , is the force given in Eq. (142) from the energy effects due to displacement, plus the flow force given in Eq. (145) plus the force due to the tension/flow effects given in Eq. (174)

$$\begin{aligned}
\mathbf{F}_L &= \mathbf{F}_{LD} + \mathbf{F}_{LF} + \mathbf{F}_{LT} = -Q\nabla\phi - Q\partial\mathbf{A}/\partial t \\
&\quad + \mathbf{i}(\partial A_{AZ}/\partial x)QU_Z + \mathbf{j}(\partial A_{AZ}/\partial y)QU_Z \\
&\quad - \mathbf{i}(\partial A_{AX}/\partial z)QU_Z - \mathbf{j}(\partial A_{AY}/\partial z)QU_Z \\
&\quad + \mathbf{j}(\partial A_{AX}/\partial y)QU_X + \mathbf{k}(\partial A_{AX}/\partial z)QU_X \\
&\quad - \mathbf{j}(\partial A_{AY}/\partial x)QU_X - \mathbf{k}(\partial A_{AZ}/\partial x)QU_X \\
&\quad + \mathbf{k}(\partial A_{AY}/\partial z)QU_Y + \mathbf{i}(\partial A_{AY}/\partial x)QU_Y \\
&\quad - \mathbf{k}(\partial A_{AZ}/\partial y)QU_Y - \mathbf{i}(\partial A_{AX}/\partial y)QU_Y. \quad (175)
\end{aligned}$$

Now form $\nabla\mathbf{x}\mathbf{A} = [\mathbf{i}\partial/\partial x + \mathbf{j}\partial/\partial y + \mathbf{k}\partial/\partial z] \times [\mathbf{i}A_{AX} + \mathbf{j}A_{AY} + \mathbf{k}A_{AZ}] = \mathbf{k}\partial A_{AY}/\partial x - \mathbf{j}\partial A_{AZ}/\partial x - \mathbf{k}\partial A_{AX}/\partial y + \mathbf{i}\partial A_{AZ}/\partial y + \mathbf{j}\partial A_{AX}/\partial z - \mathbf{i}\partial A_{AY}/\partial z$, and then form $\mathbf{U}\mathbf{x}\nabla\mathbf{x}\mathbf{A} = U_X(-\mathbf{j}\partial A_{AY}/\partial x - \mathbf{k}\partial A_{AZ}/\partial x + \mathbf{j}\partial A_{AX}/\partial y + \mathbf{k}\partial A_{AX}/\partial z) + U_Y(\mathbf{i}\partial A_{AY}/\partial x - \mathbf{i}\partial A_{AX}/\partial y - \mathbf{k}\partial A_{AZ}/\partial y + \mathbf{k}\partial A_{AY}/\partial z) + U_Z(\mathbf{i}\partial A_{AZ}/\partial x + \mathbf{j}\partial A_{AZ}/\partial y - \mathbf{i}\partial A_{AX}/\partial z - \mathbf{j}\partial A_{AY}/\partial z)$, which allows Eq. (175) to become

$$\begin{aligned}
\mathbf{F}_L &= \mathbf{F}_{LD} + \mathbf{F}_{LF} + \mathbf{F}_{LT} \\
&= -Q\nabla\phi - Q\partial\mathbf{A}/\partial t + Q\mathbf{U}\mathbf{x}\nabla\mathbf{x}\mathbf{A}. \quad (176)
\end{aligned}$$

Substituting in Eq. (123), $\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$, and Eq. (124), $\mathbf{B} = \nabla\mathbf{x}\mathbf{A}$ leaves

$$\mathbf{F}_L = Q(\mathbf{E} + \mathbf{U}\mathbf{x}\mathbf{B}). \quad (177)$$

Equation (177) is recognized as the Lorentz force equation.

PART D. GRAVITATION

D.1. The gravitational Poisson equation

We now define ρ_E , the extrinsic-energy-density for massive bodies, as

$$\rho_E = \gamma M c^2 / V. \quad (178)$$

In Eq. (178), M is the mass in a small volume V and $\gamma = (1 - v^2/c^2)^{-1/2}$, where v is the velocity of the mass and c the speed of light, and all quantities are measured with respect to the rest frame of the aether. We now recall from Section B.4:

The extrinsic-energy force-reduction postulate. The presence of extrinsic-energy decreases the positive (negative) attached-aether tension and the negative (positive) attached-aether quantum-force by an amount proportional to the amount of extrinsic-energy present with a constant of proportionality K_{G1} (K_{G2}).

With X_Q the size of the analysis-cube, the extrinsic-energy force-reduction postulate can be expressed mathematically for the nominal analysis-cube as

$$F_{TP} = K_{TP}X_Q = K_{T0}(1 - K_{G1}\rho_E)X_Q, \quad (179)$$

$$F_{TN} = K_{TN}X_Q = K_{T0}(1 - K_{G2}\rho_E)X_Q, \quad (180)$$

$$F_{QP} = 2K_{QP}/X_Q^3 = 2K_{Q0}(1 - K_{G2}\rho_E)/X_Q^3, \quad (181)$$

$$F_{QN} = 2K_{QN}/X_Q^3 = 2K_{Q0}(1 - K_{G1}\rho_E)/X_Q^3. \quad (182)$$

From Eqs. (179) to (182), the force parameters are

$$K_{TP} = K_{T0}(1 - K_{G1}\rho_E), \quad (183)$$

$$K_{TN} = K_{T0}(1 - K_{G2}\rho_E), \quad (184)$$

$$K_{QP} = K_{Q0}(1 - K_{G2}\rho_E), \quad (185)$$

$$K_{QN} = K_{Q0}(1 - K_{G1}\rho_E). \quad (186)$$

Reference 1 shows that equilibrium between the quantum-pressure and tension leads to a cube size of

$$X_1 \approx X_0(1 + [K_{G1} - K_{G2}]\rho_E/4). \quad (192)$$

Now recall Eq. (23), $X_0 = (2K_{Q0}/K_{T0})^{1/4}$, and with density inversely proportional to volume, $\rho_{PA}/\rho_0 = (1 + [K_{G1} - K_{G2}]\rho_E/4)^{-3} \approx (1 - 3[K_{G1} - K_{G2}]\rho_E/4)$. We now define

$$\rho_G = 3[K_{G1} - K_{G2}]\rho_E\rho_0/2 \quad (193)$$

leaving

$$\rho_{PA} = \rho_0 - \rho_G/2. \quad (194)$$

A similar derivation for the negative-aether gives

$$\rho_{NA} = \rho_0 + \rho_G/2. \quad (197)$$

Figure 8 presents a diagram of what happens when extrinsic-energy is injected into positive-attached-aether. We see from Eq. (194) that this injection results in an expansion of the positive-attached-aether cube. Prior to the injection of the extrinsic-energy, the amount of positive-attached-aether is the nominal density ρ_0 multiplied by the volume of the cube, $\Delta x\Delta y\Delta z$. When we inject extrinsic-energy into that cube, the amount of positive-attached-aether will remain the same, but the volume of the cube will expand, with Δx becoming larger by δx and similar increases in Δy and Δz . The positive-attached-aether density is then the original amount of aether divided by the new larger volume, and this new density is

$$\begin{aligned}
\rho_{PA} &= \rho_0(\Delta x\Delta y\Delta z)/[(\Delta x + \delta x)(\Delta y + \delta y)(\Delta z + \delta z)] \\
&\approx \rho_0(\Delta x\Delta y\Delta z)/(\Delta x\Delta y\Delta z + \delta x\Delta y\Delta z \\
&\quad + \delta y\Delta x\Delta z + \delta z\Delta x\Delta y) \\
&= \rho_0/(1 + \delta x/\Delta x + \delta y/\Delta y + \delta z/\Delta z) \\
&\approx \rho_0(1 - \delta x/\Delta x - \delta y/\Delta y - \delta z/\Delta z). \quad (198)
\end{aligned}$$

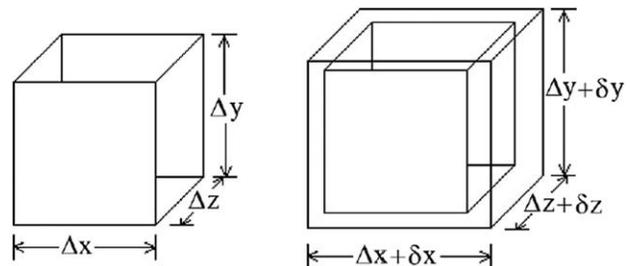


FIG. 8. Analysis-cube of undisturbed aether (left). Cube with injected extrinsic-energy (right).

We next observe that the expansion of the cube is related to the difference in the displacement vector \mathbf{P}_G between its value at the cube center and its value at the edge face of the cube, $\delta x/2 = P_{GX}(x + \Delta x/2, y, z, t) - P_{GX}(x, y, z, t)$. We then divide by $\Delta x/2$ to arrive at $\delta x/\Delta x = [P_{GX}(x + \Delta x/2, y, z, t) - P_{GX}(x, y, z, t)]/\Delta x/2 = \partial P_{GX}/\partial x$, where the last equality is in the limit when we shrink our analysis-cube to zero size. Repeating the derivation for y and z will lead to similar expressions. Hence, Eq. (198) can be re-expressed as $\rho_{PA} = \rho_0(1 - \delta x/\Delta x - \delta y/\Delta y - \delta z/\Delta z) = \rho_0(1 - \partial P_{GX}/\partial x - \partial P_{GY}/\partial y - \partial P_{GZ}/\partial z)$, or,

$$\rho_{PA} \approx \rho_0(1 - \nabla \cdot \mathbf{P}_G). \quad (199)$$

We now rearrange the terms of Eq. (199) and then substitute in the value of ρ_{PA} from Eq. (194)

$$\nabla \cdot \mathbf{P}_G \approx (\rho_0 - \rho_{PA})/\rho_0 = \rho_G/2\rho_0. \quad (200)$$

Next we observe from Fig. 8 that \mathbf{P}_G is purely longitudinal. That is, extrinsic-energy pushes outward on the cube walls but it does not cause any rotation. A purely longitudinal vector field can be formed from the gradient of a scalar field. The scalar field is named ψ_{GP} ,

$$\mathbf{P}_{GL} = \nabla \psi_{GP}. \quad (201)$$

This allows us to obtain

$$\nabla \cdot \mathbf{P}_G = \nabla \cdot \mathbf{P}_{GL} = \nabla \cdot \nabla \psi_{GP} = \nabla^2 \psi_{GP}. \quad (202)$$

Next, combine Eqs. (200) and (202) to yield

$$\nabla^2 \psi_{GP} = \nabla \cdot \mathbf{P}_{GL} = \nabla \cdot \mathbf{P}_G = \rho_G/2\rho_0. \quad (203)$$

A similar derivation can be applied to the negative-aether to arrive at

$$\nabla^2 \psi_{GN} = \nabla \cdot \mathbf{N}_{GL} = \nabla \cdot \mathbf{N}_G = -\rho_G/2\rho_0. \quad (205)$$

Subtract Eq. (205) from Eq. (203)

$$\nabla^2(\Psi_{GP} - \Psi_{GN}) = \rho_G/\rho_0. \quad (207)$$

Now define ϕ_G by $\phi_G = -(\Psi_{GP} - \Psi_{GN})\rho_0/\varepsilon_0$, where ε_0 is the permittivity of free space. This results in Eq. (207) becoming Poisson's equation

$$\nabla^2 \phi_G = -\rho_G/\varepsilon_0. \quad (208)$$

With this definition for ϕ_G , we can also derive

$$\mathbf{P}_{GL} - \mathbf{N}_{GL} = \nabla(\Psi_{GP} - \Psi_{GN}) = -\varepsilon_0 \nabla \phi_G/\rho_0. \quad (209)$$

We see above that the presence of extrinsic-energy leads to displacement of the negative-attached-aether that is always equal and opposite to the displacement of the positive-attached-aether. Since injection of extrinsic-energy is the only physical cause for the longitudinal displacements \mathbf{P}_{GL} and \mathbf{N}_{GL} of the attached-aether, we arrive at

$$\mathbf{N}_{GL} = -\mathbf{P}_{GL}. \quad (211)$$

D.2. The gamma force

To better understand the force caused by the extrinsic-energy, it is useful to consider the case of a sphere of extrinsic-energy injected into the attached-aether. The solutions to Eq. (208) (Poisson's equation) for a uniform sphere of extrinsic-energy with radius R_0 are

$$\begin{aligned} \mathbf{P}_{GIN} &= (\rho_G/6\rho_0)(xi + yj + zk) \\ &= (\rho_G/6\rho_0)\mathbf{r} \quad (r < R_0), \end{aligned} \quad (212)$$

$$\mathbf{P}_{GOUT} = P_0 R_0^2 \hat{\mathbf{r}}/r^2 = (\rho_G/6\rho_0)R_0^3 \hat{\mathbf{r}}/r^2 \quad (r > R_0). \quad (213)$$

We see from Eq. (212) that the analytic-cubes will expand inside a source. With an expanded central spherical region, concentric spherical shells of positive-attached-aether inside the sphere will move outward, also getting thicker. Outside of the sphere concentric spherical shells will also move outward, this time thinning and having increased surface areas. It is the force from the extrinsic-energy that leads to the displacements both inside and outside of the sphere of extrinsic-energy, as the extrinsic-energy is pushing the positive-attached-aether radially outward in both regions.

On the walls of a quantum-cube, Eq. (15) gives the inward force due to tension as $F_{T0} = \text{ld}E_T/\text{d}X_Q = K_{T0}X_Q$ and Eq. (16) gives the outward force due to quantum-pressure as $F_{Q0} = \text{ld}E_Q/\text{d}X_Q = 2K_{Q0}/X_Q^3$. In Section D.1, we have seen that injection of extrinsic-energy changes the size of a positive-attached-aether quantum. Equations (15) and (16) indicate that the increased size will increase the tension and decrease the quantum-pressure. To maintain the larger size X_Q , the presence in the cube of extrinsic-energy must therefore result in an extrinsic-energy-immersion force \mathbf{F}_γ (the gamma-force) that acts on the walls of the positive-attached-aether analysis-cube that is equal and opposite to the sum of the longitudinal-tension-force and the quantum-pressure force

$$\mathbf{F}_\gamma = -\mathbf{F}_{TL} - \mathbf{F}_Q. \quad (216)$$

Equation (216) specifies the definition of the gamma force. Note that we use the longitudinal component of the tension force because only the longitudinal component is supplied by the extrinsic-energy. When a source of extrinsic-energy is present, the cubes are displaced and distorted both inside and outside of the source regions. The gamma force therefore exists both inside and outside the source regions. The gamma force is the force which balances the sum of forces on all cube faces for gravitational effects. The gamma force consists of two components:

$$\mathbf{F}_{\gamma PT} = 4K_c(1 + K_{GC})K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}, \quad (218)$$

$$\mathbf{F}_{\gamma PQ} = -4K_c K_{GC} K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}. \quad (219)$$

In Eqs. (218) and (219), K_{GC} is a coupling constant.

D.3. Work done on displaced aether

When a cube of aether moves through the tension, quantum-pressure and gamma force fields, work is done as described in Eqs. (32) and (33). The work evaluation is quite tedious, and here we will only present the results (see Ref. 1 for details). The work done against tension inside of a sphere of extrinsic-energy when a cube moves a distance \mathbf{P}_G due to that extrinsic-energy is

$$E_{\text{TPGI}} = K_{\text{T0}}X_0^2 \left[(1/2) + K_c |\mathbf{P}_G|/\xi_0 + K_c^2 (|\mathbf{P}_G|/\xi_0)^2 + 3\delta X/X_0 + 3K_c |\mathbf{P}_G| \delta X/2X_0 \xi_0 \right]. \quad (214)$$

The work done against the quantum-pressure inside of a sphere of extrinsic-energy for a cube displacement \mathbf{P}_G due to extrinsic-energy is

$$E_{\text{QPGI}} = (2K_{\text{Q0}}/X_0^2) \left[(1/2) - K_c |\mathbf{P}_G|/\xi_0 + K_c^2 |\mathbf{P}_G|^2/\xi_0^2 - 3\delta X/X_0 + 3K_c |\mathbf{P}_G| \delta X/2X_0 \xi_0 \right]. \quad (215)$$

The work done against the gamma force inside of a sphere of extrinsic-energy for a cube displacement \mathbf{P}_G due to extrinsic-energy is

$$E_{\gamma\text{PGI}} = K_{\text{T0}}X_0^2 \left[-2K_c^2 (|\mathbf{P}_G|/\xi_0)^2 - 4K_c^2 K_{\text{GC}} (|\mathbf{P}_G|/\xi_0)^2 + 3K_c |\mathbf{P}_G| \delta X/X_0 \xi_0 \right]. \quad (224)$$

The total work is the sum of Eqs. (214), (215), and (224)

$$E_{\text{PG}} = K_{\text{T0}}X_0^2 (1 - 4K_c^2 K_{\text{GC}} |\mathbf{P}_G|^2/\xi_0^2). \quad (225)$$

A similar expression is derived for the negative-attached-aether

$$E_{\text{NG}} = K_{\text{T0}}X_0^2 (1 - 4K_c^2 K_{\text{GC}} |\mathbf{N}_G|^2/\xi_0^2). \quad (226)$$

D.4. Newtonian gravity

We will now use Eqs. (225) and (226) to calculate the force that will occur on a sphere of extrinsic-energy in the presence of an ambient aetherial displacement $\mathbf{P}_{\text{AG}} = -\mathbf{N}_{\text{AG}}$. We assert that we can build any arbitrary distribution of extrinsic-energy from small enough uniform spheres, and that the superposition of the resulting fields will allow for the calculation of the general situation.

To analyze what happens to a sphere of extrinsic-energy in the presence of an ambient aetherial displacement, we will first divide the sphere into slices, and then divide the slices into strips as shown in Fig. 9. Then we will look at small cubes within those strips. The force on each cube will be evaluated, and we will sum up the force on all of the cubes to find the total force on the sphere.

If there is no ambient displacement of the aether, the extrinsic-energy will push the attached-aether out radially. That displacement will be zero at the center of the sphere

and increase radially until the edge, and this is a solution of Eq. (208)

$$\mathbf{P}_{\text{GS}} = (\rho_G/6\rho_0)\mathbf{r} = (\rho_G/6\rho_0)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}). \quad (227)$$

Equation (227) is of course just Eq. (212) but now we use \mathbf{P}_{GS} for the displacement inside the sphere.

The ambient attached-aether displacement is \mathbf{P}_{AG} , which without loss of generality is considered to be in the x direction, $\mathbf{P}_{\text{AG}} = P_{\text{AG}}\mathbf{i}$. We then get the total attached-aether displacement within the sphere of $\mathbf{P}_G = \mathbf{P}_{\text{AG}} + \mathbf{P}_{\text{GS}} = [P_{\text{AG}} + (\rho_G/6\rho_0)x]\mathbf{i} + (\rho_G/6\rho_0)y\mathbf{j} + (\rho_G/6\rho_0)z\mathbf{k}$.

Returning to Fig. 9, within the strip, and using Eqs. (225) and (227), the energy of a small positive-attached-aether analysis-cube centered at x, y, z is

$$\begin{aligned} E_{\text{PG}}(x, y, z) &= K_{\text{T0}}X_0^2 (1 - 4K_c^2 K_{\text{GC}} |\mathbf{P}_{\text{AG}} + \mathbf{P}_{\text{GS}}|^2/\xi_0^2) \\ &= K_{\text{T0}}X_0^2 \{ 1 - 4K_c^2 K_{\text{GC}} [P_{\text{AG}}^2 \\ &\quad + (\rho_G/6\rho_0)x]^2 + (\rho_G/6\rho_0)y^2 + (\rho_G/6\rho_0)z^2]/\xi_0^2 \} \\ &= K_{\text{T0}}X_0^2 \{ 1 - 4K_c^2 K_{\text{GC}} [P_{\text{AG}}^2 + 2P_{\text{AG}}(\rho_G/6\rho_0)x \\ &\quad + (\rho_G/6\rho_0)^2 x^2 + (\rho_G/6\rho_0)^2 y^2 + (\rho_G/6\rho_0)^2 z^2]/\xi_0^2 \}. \end{aligned} \quad (228)$$

We then calculate the effect of a virtual displacement δx and that gives us

$$\begin{aligned} E_{\text{PG}}(x + \delta x, y, z) &= K_{\text{T0}}X_0^2 \{ 1 - 4K_c^2 K_{\text{GC}} [P_{\text{AG}}^2 + 2P_{\text{AG}}(\rho_G/6\rho_0)(x + \delta x) \\ &\quad + (\rho_G/6\rho_0)^2 (x + \delta x)^2 + (\rho_G/6\rho_0)^2 y^2 \\ &\quad + (\rho_G/6\rho_0)^2 z^2]/\xi_0^2 \}. \end{aligned} \quad (229)$$

Subtracting Eq. (228) from Eq. (229) leaves the energy change during the virtual displacement

$$\begin{aligned} \delta E_{\text{PG}}(x, y, z) &= E_{\text{PG}}(x + \delta x, y, z) - E_{\text{PG}}(x, y, z) \\ &= -4K_{\text{T0}}X_0^2 K_c^2 K_{\text{GC}} \\ &\quad \left[2P_{\text{AG}}(\rho_G/6\rho_0)\delta x + (\rho_G/6\rho_0)^2 (2x\delta x + \delta x^2) \right] / \xi_0^2. \end{aligned} \quad (230)$$

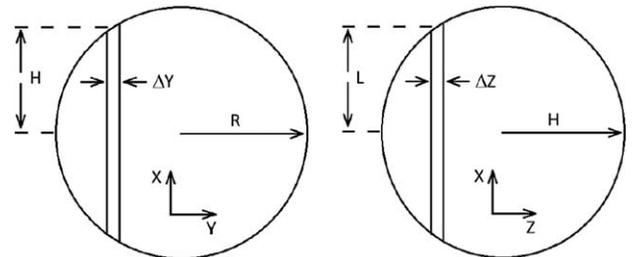


FIG. 9. A sphere of extrinsic-energy showing a slice of width ΔY (left). A slice of the sphere showing a strip of depth ΔZ (right).

In the above expression, we can drop the term that is second order in the small quantity δx , as we will take the limit as $\delta x \rightarrow 0$. We can now evaluate the force on the strip by considering the sum of all volume elements within the strip. The term $-4K_{T0}X_0^2K_c^2K_{GC}(\rho_G/6\rho_0)^2(2x\delta x)/\xi_0^2$ can be dropped because for every value of positive x in our strip there is a value of negative x of equal magnitude. The surviving term is $-4K_{T0}X_0^2K_c^2K_{GC}2P_{AG}(\rho_G/6\rho_0)\delta x/\xi_0^2$, and this term is independent of x , y , or z . Recalling that Eq. (230) refers to the change in energy for a single analysis-cube within the strip, we can form the relation for the force on the whole strip by summing over all of the analysis-cubes within the strip (Σ_{strip} is the symbol for that sum). The volume of the strip is $2L\Delta y\Delta z$, and therefore, the number of analysis-cubes within the strip is $2L\Delta y\Delta z/X_0^3$, and the magnitude of the force on the strip is

$$\begin{aligned} F_{\text{stripPG}} &= \Sigma_{\text{strip}} \{ \delta \Delta E_{\text{PG}}(x, y, z) / \delta x \} \\ &= \left[4K_{T0}X_0^2K_c^2K_{GC}2P_{AG}(\rho_G/6\rho_0) / \xi_0^2 \right] \\ &\quad [2L\Delta y\Delta z/X_0^3] \\ &= 8L\Delta y\Delta zK_{T0}K_c^2K_{GC}P_{AG}\rho_G/3\rho_0X_0\xi_0^2. \end{aligned} \quad (231)$$

The force on the strip shown in Fig. 9 is proportional to the volume of the strip ($2L\Delta y\Delta z$) but independent of x , y , and z . The sum of the volume of all of the strips will be the volume of the sphere, V_{sphere} . Hence, we can sum the forces from all such strips to arrive at

$$F_{\text{spherePG}} = 4V_{\text{sphere}}K_{T0}K_c^2K_{GC}P_{AG}\rho_G/3\rho_0X_0\xi_0^2. \quad (232)$$

Including the force from the negative aether will lead to a factor of two increase in the force, and here, the force direction is included by replacing P_{AG} with $-\mathbf{P}_{AG}$ so the total force is

$$\mathbf{F}_{\text{sphereG}} = -8V_{\text{sphere}}K_{T0}K_c^2K_{GC}\mathbf{P}_{AG}\rho_G/3\rho_0X_0\xi_0^2. \quad (234)$$

We now recall Eq. (209), $\mathbf{P}_{GL} - \mathbf{N}_{GL} = -\varepsilon_0\nabla\phi_G/\rho_0$, and recall that before the injection of the extrinsic-energy $\mathbf{P}_{GL} = \mathbf{P}_{AG}$. And now with Eq. (211) $\mathbf{N}_{GL} = -\mathbf{P}_{GL}$ we have $\mathbf{P}_{GL} - \mathbf{N}_{GL} = 2\mathbf{P}_{GL} = 2\mathbf{P}_{AG} = -\varepsilon_0\nabla\phi_G/\rho_0$, or $\mathbf{P}_{AG} = -\varepsilon_0\nabla\phi_G/2\rho_0$. Thus, we arrive at

$$\mathbf{F}_{\text{sphereG}} = 4V_{\text{sphere}}K_{T0}K_c^2K_{GC}\varepsilon_0\nabla\phi_G\rho_G/3\rho_0^2X_0\xi_0^2. \quad (235)$$

We now recall $K_{T0} = 3\rho_0^2X_0\xi_0^2/4K_c^2\varepsilon_0$ from Eq. (141) and we also set ρ_G multiplied by the volume of the sphere multiplied by K_{GC} to a quantity called Q_G ,

$$Q_G = K_{GC}V_{\text{sphere}}\rho_G. \quad (236)$$

We then arrive at the force on an amount of extrinsic-energy immersed within a region of ambient attached-aether displacement

$$\mathbf{F}_G = Q_G\nabla\phi_G. \quad (237)$$

We will now evaluate the gravitational potential of a uniform sphere of mass. In that case, we have $(4/3)\pi R_S^3\rho_{ES} = \gamma M_S c^2$, or $\rho_{ES} = 3\gamma M_S c^2/4\pi R_S^3$, where R_S is the radius of the sphere, M_S is the mass of the sphere, and ρ_{ES} is the extrinsic-energy density inside of the sphere. Recalling Eq. (193), $\rho_G = 3[K_{G1} - K_{G2}]\rho_E\rho_0/2$, we can define ρ_{GS} to be related to ρ_{ES} as

$$\begin{aligned} \rho_{GS} &= 3[K_{G1} - K_{G2}]\rho_{ES}\rho_0/2 \\ &= 9[K_{G1} - K_{G2}]\gamma M_S c^2\rho_0/8\pi R_S^3 \quad (r < R_S). \end{aligned} \quad (238)$$

Recall Eq. (208), $\nabla^2\phi_G = -\rho_G/\varepsilon_0$. For the case of spherical symmetry, the derivatives with respect to the angular variables vanish leaving Eq. (208) as

$$\partial^2\phi_G/\partial\rho^2 + (2/r)\partial\phi_G/\partial r = -\rho_G/\varepsilon_0. \quad (239)$$

The analysis will be divided into two regions, one inside of R_S where $\phi_G = \phi_{SI}$ and the other outside of R_S where $\phi_G = \phi_{SO}$. Inside of R_S the solution to Eq. (239) is

$$\phi_{SI} = -\rho_{GS}r^2/6\varepsilon_0. \quad (240)$$

Now form $\nabla\phi_{SI}$,

$$\nabla\phi_{SI} = -\rho_{GS}\mathbf{r}/3\varepsilon_0. \quad (241)$$

We again recall Eq. (209), $\mathbf{P}_{GL} - \mathbf{N}_{GL} = -\varepsilon_0\nabla\phi_G/\rho_0$ and Eq. (211), $\mathbf{N}_{GL} = -\mathbf{P}_{GL}$, and hence $\mathbf{P}_{GL} - \mathbf{N}_{GL} = 2\mathbf{P}_{GL} = -\varepsilon_0\nabla\phi_G/\rho_0$, or

$$\mathbf{P}_{GL} = -\varepsilon_0\nabla\phi_G/2\rho_0. \quad (242)$$

Since ϕ_{SI} is a specific form of ϕ_G ,

$$\mathbf{P}_{GLIN} = -\varepsilon_0\nabla\phi_{SI}/2\rho_0 = \rho_{GS}\mathbf{r}/6\rho_0 = -\mathbf{N}_{GLIN}. \quad (243)$$

And now use Eq. (238) for ρ_{GS} ,

$$\begin{aligned} \mathbf{P}_{GLIN} &= -\mathbf{N}_{GLIN} = ([K_{G1} - K_{G2}]\rho_{ES}/4)\mathbf{r} \\ &= (3[K_{G1} - K_{G2}]\gamma M_S c^2/16\pi R_S^3)\mathbf{r}. \end{aligned} \quad (244)$$

Outside of R_S , ρ_M and ρ_G are zero, and the solution to Eq. (239) is

$$\phi_{SO} = \rho_{GS}R_S^3/3r\varepsilon_0 - \rho_{GS}R_S^2/2\varepsilon_0. \quad (246)$$

Equation (246) involves the density ρ_{GS} inside the sphere because it sets the boundary condition. The density ρ_G outside the sphere is zero. Equation (246) leads to

$$\nabla\phi_{SO} = -(\rho_{GS}R_S^3/3r^2\varepsilon_0)\hat{\mathbf{r}}. \quad (247)$$

Now use Eq. (242), $\mathbf{P}_{GL} = -\varepsilon_0\nabla\phi_G/2\rho_0$, where now ϕ_{SO} is a specific form of ϕ_G ,

$$\begin{aligned} \mathbf{P}_{GLOUT} &= -\varepsilon_0\nabla\phi_{SO}/2\rho_0 \\ &= (\rho_{GS}R_S^3/6r^2\rho_0)\hat{\mathbf{r}} = -\mathbf{N}_{GLOUT}. \end{aligned} \quad (248)$$

Using Eq. (238) for ρ_{GS} leaves

$$\mathbf{P}_{GLOUT} = -\mathbf{N}_{GLOUT} = (3[K_{G1} - K_{G2}]\gamma M_S c^2 / 16\pi r^2) \hat{\mathbf{r}}. \quad (249)$$

Note that Eqs. (244) and (249) obtain equal values at $r = R_S$, as they must.

It is possible to further manipulate Eq. (237) into a more familiar form for the case of two interacting masses. Consider two homogenous spheres S_1 and S_2 , with masses M_1 and M_2 , respectively, and radii R_1 and R_2 , respectively. Without loss of generality, we can consider S_2 to be centered at the origin. With S_1 centered at $r > R_1 + R_2$, Eq. (247) informs that $\nabla\phi_{SO2}$ from M_2 is $\nabla\phi_{SO2} = -(\rho_{GS2} R_2^3 / 3r^2 \varepsilon_0) \hat{\mathbf{r}}$ and therefore Eq. (237) for the force between the two masses becomes $\mathbf{F}_{GM1M2} = Q_{G1} \nabla\phi_{SO2} = -(Q_{G1} \rho_{GS2} R_2^3 / 3r^2 \varepsilon_0) \hat{\mathbf{r}}$. Recall Eq. (238), $\rho_{GS} = 3[K_{G1} - K_{G2}] \rho_{ES} \rho_0 / 2$, to get $\mathbf{F}_{GM1M2} = Q_{G1} \nabla\phi_{SO2} = -(Q_{G1} [K_{G1} - K_{G2}] \rho_{E2} R_2^3 \rho_0 / 2r^2 \varepsilon_0) \hat{\mathbf{r}}$. And with $\rho_{E2} = \gamma_2 M_2 c^2 / [(4/3)\pi R_2^3]$ this becomes $\mathbf{F}_{GM1M2} = -(3\gamma_2 M_2 c^2 / 8\pi r^2 \varepsilon_0) Q_{G1} [K_{G1} - K_{G2}] \rho_0 / 2$. Next, recall from above Eq. (236) that Q_G is $K_{GC} V_{\text{sphere}} \rho_G$, or, in this case, using Eq. (193), $Q_{G1} = K_{GC} (4/3)\pi R_1^3 \rho_{G1} = K_{GC} 2\pi R_1^3 [K_{G1} - K_{G2}] \rho_{E1} \rho_0$. With $\rho_{E1} = \gamma_1 M_1 c^2 / [(4/3)\pi R_1^3]$, $Q_{G1} = 3K_{GC} [K_{G1} - K_{G2}] \rho_0 \gamma_1 M_1 c^2 / 2$, leaving

$$\begin{aligned} \mathbf{F}_{ME1M2} &= \mathbf{F}_{GNEWTON} \\ &= -(9\gamma_2 M_2 c^2 K_{GC} [K_{G1} - K_{G2}]^2 \rho_0^2 \gamma_1 M_1 c^2 / 16\pi r^2 \varepsilon_0) \hat{\mathbf{r}} \\ &= -(G_N \gamma_1 M_1 \gamma_2 M_2 / r^2) \hat{\mathbf{r}}. \end{aligned} \quad (250)$$

In the low velocity limit, $\gamma_1 = \gamma_2 = 1$, and Eq. (250) is recognized as Newton's Law of Universal Gravitation where G_N is the combination of constants

$$\begin{aligned} G_N &= 9K_{GC} [K_{G1} - K_{G2}]^2 \rho_0^2 c^4 / 16\pi \varepsilon_0 \\ &= 6.6743 \times 10^{-11} \text{Nm}^2/\text{kg}^2. \end{aligned} \quad (251)$$

D.5. Field-masses

Here, we repeat Eqs. (214), (215), and (224) from Section D.3

$$\begin{aligned} E_{TPGI} &= K_{T0} X_0^2 \left[(1/2) + K_c |\mathbf{P}_G| / \xi_0 + K_c^2 (|\mathbf{P}_G| / \xi_0)^2 \right. \\ &\quad \left. + 3\delta X / X_0 + 3K_c |\mathbf{P}_G| \delta X / 2X_0 \xi_0 \right], \end{aligned} \quad (214)$$

$$\begin{aligned} E_{QPGI} &= (2K_{Q0} / X_0^2) \left[(1/2) - K_c |\mathbf{P}_G| / \xi_0 \right. \\ &\quad \left. + K_c^2 |\mathbf{P}_G|^2 / \xi_0^2 - 3\delta X / X_0 + 3K_c |\mathbf{P}_G| \delta X / 2X_0 \xi_0 \right], \end{aligned} \quad (215)$$

$$\begin{aligned} E_{\gamma PGI} &= K_{T0} X_0^2 \left[-2K_c^2 (|\mathbf{P}_G| / \xi_0)^2 - 4K_c^2 K_{GC} (|\mathbf{P}_G| / \xi_0)^2 \right. \\ &\quad \left. + 3K_c |\mathbf{P}_G| \delta X / X_0 \xi_0 \right]. \end{aligned} \quad (224)$$

The above equations are for energies inside of a sphere of mass. Outside of a sphere of mass we derive similar equations in Ref. 1

$$\begin{aligned} E_{TPGO} &= K_{T0} X_0^2 \left[(1/2) + K_c |\mathbf{P}_G| / \xi_0 \right. \\ &\quad \left. + K_c^2 (|\mathbf{P}_G| / \xi_0)^2 - K_c |\mathbf{P}_G| \delta X / 4X_0 \xi_0 \right], \end{aligned} \quad (H2)$$

$$\begin{aligned} E_{QPGO} &= (2K_{Q0} / X_0^2) \left[(1/2) - K_c |\mathbf{P}_G| / \xi_0 \right. \\ &\quad \left. + K_c^2 (|\mathbf{P}_G| / \xi_0)^2 - K_c |\mathbf{P}_G| \delta X / 4X_0 \xi_0 \right], \end{aligned} \quad (H4)$$

$$\begin{aligned} E_{\gamma PGO} &= K_{T0} X_0^2 \left[-2K_c^2 (|\mathbf{P}_G| / \xi_0)^2 - 4K_c^2 K_{GC} (|\mathbf{P}_G| / \xi_0)^2 \right. \\ &\quad \left. + K_c |\mathbf{P}_G| \delta X / 2X_0 \xi_0 \right]. \end{aligned} \quad (H6)$$

We see that the portion of these equations not involving $\delta X / X_0$ are identical, and since $\delta X / X_0$ is assumed small, we drop the $\delta X / X_0$ terms to arrive at equations which we will now use both inside and outside of a sphere of mass

$$E_{TPG} = (K_{T0} X_0^2) \left[(1/2) + K_c |\mathbf{P}_G| / \xi_0 + K_c^2 |\mathbf{P}_G|^2 / \xi_0^2 \right], \quad (253)$$

$$E_{QPG} = (2K_{Q0} / X_0^2) \left[(1/2) - K_c |\mathbf{P}_G| / \xi_0 + K_c^2 |\mathbf{P}_G|^2 / \xi_0^2 \right], \quad (254)$$

$$\begin{aligned} E_{\gamma PG} &= -2K_{T0} K_c^2 (X_0 / \xi_0)^2 |\mathbf{P}_G|^2 \\ &\quad - 4K_c^2 K_{GC} K_{T0} (X_0 / \xi_0)^2 |\mathbf{P}_G|^2. \end{aligned} \quad (255)$$

At this point, we will now introduce the empirical *Gravitational-Mass Assignment*:

The Gravitational-Mass Assignment. The tension-energy E_{TPG} (E_{TNG}) leads to a positive (negative) gravitational-mass while the quantum-energy E_{QPG} (E_{QNG}) leads to a negative (positive) gravitational-mass.

It is shown in Ref. 1 that the second term of the gamma energy comes half from a tension component and half from a quantum-pressure component and so its contribution to gravitational mass vanishes, while the first term in the gamma energy is a negative tension in the positive-attached-aether. Summing all of the masses given by the gravitational mass assignment then results in the following equation for the total gravitational mass of the tension, quantum-pressure, and gamma field energies in the positive-attached-aether

$$m_{PTQ\gamma} = 2(K_{T0} X_0^2 / c^2) \left[K_c |\mathbf{P}_G| / \xi_0 - K_c^2 |\mathbf{P}_G|^2 / \xi_0^2 \right]. \quad (256)$$

Similarly, for the negative-attached-aether, we obtain

$$m_{NTQ\gamma} = 2(K_{T0} X_0^2 / c^2) \left[K_c |\mathbf{N}_G| / \xi_0 - K_c^2 |\mathbf{N}_G|^2 / \xi_0^2 \right]. \quad (260)$$

Since \mathbf{N}_G and \mathbf{P}_G are purely longitudinal and with Eq. (211), $\mathbf{N}_{GL} = -\mathbf{P}_{GL}$, we get $|\mathbf{N}_G| = |\mathbf{P}_G|$ and the total field-mass is therefore

$$\begin{aligned} m_{TQ\gamma} &= m_{PTQ\gamma} + m_{NTQ\gamma} \\ &= 4(K_{T0} X_0^2 / c^2) \left[K_c |\mathbf{P}_G| / \xi_0 - K_c^2 |\mathbf{P}_G|^2 / \xi_0^2 \right]. \end{aligned} \quad (261)$$

Next we divide by the volume of our analytic cell, X_0^3 , and separate out two field-mass densities

$$\delta\rho_{M1} = 4K_{T0}K_c|\mathbf{P}_G|/\xi_0X_0c^2 = K_{G3}P_G, \quad (262)$$

(*see note after Eq. (263))

$$\delta\rho_{M2} = -4K_c^2K_{T0}|\mathbf{P}_G|^2/X_0\xi_0^2c^2 = -K_{G4}P_G^2. \quad (263)$$

[*Note that we must neglect any first-field-mass contribution to \mathbf{P}_G when using Eq. (262) to find the first-field-mass density.]

Now we will evaluate the effects of field-mass on the force between two objects for the important case where the mass of one object is much greater than the mass of the other. We will use a coordinate system where r equals zero in the center of the large mass. The large mass will be a sphere of mass M and radius R , and the small mass will be a sphere of mass m .

Equation (262) gives $\delta\rho_{M1} = K_{G3}P_G$ as a positive field-mass-density. For $r < R$, Eq. (244) informs that $\mathbf{P}_{GLIN} = (3[K_{G1} - K_{G2}]\gamma_M s c^2/16\pi R_S^3)\mathbf{r}$, and therefore, recalling that P_{GLIN} is the magnitude of \mathbf{P}_{GLIN} , $\delta\rho_{M1} = K_{G3}P_{GLIN} = K_{G3}(3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi R^3)r$ for $r < R$. The total first-field-mass ΔM_{1IN} within a sphere of radius r can be found by integrating $\delta\rho_{M1}$ within that sphere, $\Delta M_{1IN} = 4\pi \int \delta\rho_{M1} r^2 dr = 4\pi \int K_{G3}(3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi R^3)r^3 dr$, leaving

$$\begin{aligned} \Delta M_{1IN} &= K_{G3}(3[K_{G1} - K_{G2}]\gamma_M M c^2/16R^3)r^4 \\ &= K_{G5}(\gamma_M M/R^3)r^4 \quad (\text{for } r < R). \end{aligned} \quad (266)$$

In Eq. (266), K_{G5} is a combination of other constants

$$K_{G5} = K_{G3}(3[K_{G1} - K_{G2}]c^2/16). \quad (267)$$

For $r > R$, Eq. (249) informs that $\mathbf{P}_{GLOUT} = (3[K_{G1} - K_{G2}]\gamma_M s c^2/16\pi r^2)\mathbf{r}$, and therefore, $\delta\rho_{M1} = K_{G3}P_{GLOUT} = 3K_{G3}[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi r^2$ for $r > R$. The total first-field-mass ΔM_{1SHELL} within a spherical shell of outer radius r and inner radius R can be found by integrating $\delta\rho_{M1}$ within that shell, $\Delta M_{1SHELL} = 4\pi \int \delta\rho_{M1} r^2 dr = 4\pi \int (3K_{G3}[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi r^2)r^2 dr = (3K_{G3}[K_{G1} - K_{G2}]\gamma_M M c^2/4)r$ evaluated between R and r , $\Delta M_{1SHELL} = 4K_{G5}\gamma_M M r$. Evaluating between R and r , $\Delta M_{1SHELL} = 4K_{G5}\gamma_M M(r - R)$. To get the full mass inside of r , but outside of R , we must also add the mass inside of R by evaluating Eq. (266) at R to get

$$\begin{aligned} \Delta M_{1OUT} &= 4K_{G5}\gamma_M M(r - R) + K_{G5}\gamma_M MR \\ &= 4K_{G5}\gamma_M Mr - 3K_{G5}\gamma_M MR \quad (\text{for } r > R). \end{aligned} \quad (268)$$

Equation (263) gives $\delta\rho_{M2} = -K_{G4}P_G^2$ as a negative field-mass-density. For $r < R$, Eq. (244) informs that $\mathbf{P}_{GLIN} = (3[K_{G1} - K_{G2}]\gamma_M s c^2/16\pi R_S^3)\mathbf{r}$, and therefore, recalling that P_{GLIN} is the magnitude of \mathbf{P}_{GLIN} , $\delta\rho_{M2} = -K_{G4}P_{GLIN}^2 = -K_{G4}(3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi R^3)^2 r^2$ for $r < R$. The total second-field-mass within a sphere of radius r , ΔM_{2IN} , is found by integrating $\delta\rho_{M2}$ within the sphere,

$$\Delta M_{2IN} = 4\pi \int \delta\rho_{M2} r^2 dr = 4\pi \int -K_{G4} (3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi R^3)^2 r^4 dr = -4\pi K_{G4}(3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi R^3)^2 r^5/5, \text{ or}$$

$$\begin{aligned} \Delta M_{2IN} &= -(4\pi K_{G4}/5)(3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi R^3)^2 r^5 \\ &= -K_{G6}(\gamma_M M/R^3)^2 r^5 \quad (\text{for } r < R). \end{aligned} \quad (269)$$

In Eq. (269), K_{G6} is a combination of other constants

$$\begin{aligned} K_{G6} &= (4\pi K_{G4}/5)(3[K_{G1} - K_{G2}]c^2/16\pi)^2 \\ &= 9K_{G4}[K_{G1} - K_{G2}]^2 c^4/320\pi. \end{aligned} \quad (270)$$

For $r > R$, Eq. (249) informs that $\mathbf{P}_{GLOUT} = (3[K_{G1} - K_{G2}]\gamma_M s c^2/16\pi r^2)\mathbf{r}$, and therefore, $\delta\rho_{M2} = -K_{G4}P_G^2 = -K_{G4}P_{GLOUT}^2 = -K_{G4}(3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi r^2)^2 r^2$ for $r > R$. The total second-field-mass ΔM_{2SHELL} within a spherical shell of outer radius r and inner radius R can be found by integrating $\delta\rho_{M2}$ within that shell, $\Delta M_{2SHELL} = 4\pi \int \delta\rho_{M2} r^2 dr = -4\pi \int K_{G4}(3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi r^2)^2 r^2 dr = 4\pi K_{G4}(3[K_{G1} - K_{G2}]\gamma_M M c^2/16\pi)^2/r = 5K_{G6}(\gamma_M M)^2/r$ evaluated between R and r , and we must also add the mass from Eq. (269) evaluated at R leaving $\Delta M_{2OUT} = 5K_{G6}(\gamma_M M)^2/R - 5K_{G6}(\gamma_M M)^2/R - K_{G6}(\gamma_M M)^2/R$, or

$$\Delta M_{2OUT} = 5K_{G6}(\gamma_M M)^2/r - 6K_{G6}(\gamma_M M)^2/R \quad (\text{for } r > R). \quad (271)$$

Starting with Newtonian gravity, Eq. (250), and then adding the first and second field-masses to M for the case when $r < R$, we obtain the force on m :

$$\begin{aligned} \mathbf{F}_{GIN} &= -(G_N \gamma_m m \gamma_M M/R^3) \\ &\quad [r + K_{G5}r^2 - K_{G6}(\gamma_M M/R^3)r^3]\hat{\mathbf{r}} \quad (m \ll M; r < R). \end{aligned} \quad (272)$$

Equation (272) is relevant to the core of a galaxy when we look at an individual star within the core. In this case, the whole galactic core is the larger mass, and the individual star is the smaller mass.

Again starting with Newtonian gravity, Eq. (250), and then adding the first and second field-masses to M for the case when $r > R$, we define an effective mass M_{EFF} and get the force on m ,

$$\begin{aligned} M_{EFF} &= \gamma_M M - 3K_{G5}\gamma_M MR - 6K_{G6}\gamma_M^2 M^2/R, \quad (274) \\ \mathbf{F}_{GOUT} &= -G_N \gamma_m m (M_{EFF}/r^2 + 4K_{G5}\gamma_M M/r \\ &\quad + 5K_{G6}\gamma_M^2 M^2/r^3)\hat{\mathbf{r}} \quad (m \ll M; r > R). \end{aligned} \quad (275)$$

D.6. Dark matter

Consider a force on a star in a spiral galaxy far from the galactic core. With the mass of the galactic core much greater than the mass of the star, Eq. (275) applies. We can see that at large r , the term proportional to $1/r$ eventually dominates. Equating the centrifugal force mv^2/r to a gravitational force proportional to $1/r$ we see that the velocity of

distant stellar orbits will be independent of r , consistent with observations. Presently, this constant velocity is believed to indicate the existence of a substance called dark matter. Since the $1/r$ term in Eq. (275) was derived from the first-field-mass, we see that the first-field-mass is a major contributor to what is now known as dark matter.

Further observations of dark matter come from ultra diffuse galaxies, or UDGs. Some UDGs have very little dark matter compared to most galaxies.²⁰ This can be readily understood if some UDGs have no large central mass such as what is now called a black hole. If there is a large central mass, then Eq. (275) applies and there will be a large amount of dark matter. If there is no large central mass, then Eq. (272) applies and there will be very little dark matter.

Equation (268) gives a first-field-mass that grows with radius if there is a large galactic central mass. If this scaling were to go on indefinitely, the dark mass associated with such galaxies would become very large indeed. However, here we recall that Eq. (268) only applies if one mass is much smaller than the other mass. For the case of two neighboring galaxies, this condition no longer holds. Also, between such galaxies there will be a location where the aetherial displacement P_G is zero, and by Eq. (262) we see that the first-field-mass will be zero at such a location. Therefore, the first-field-mass will not continue to increase outside of galaxies. It will instead appear within them, but be diminished between them, which is again consistent with observations.

Neglecting the $1/r^3$ term, we will now set mv^2/r equal to the force of Eq. (275) to solve for K_{G5} . With $\gamma_m = \gamma_M = 1$ and $M_{\text{EFF}} = M$, $mv^2/r = G_N m M (1/r^2 + 4K_{G5}/r)$, or $v^2 = G_N M (1/r + 4K_{G5})$, or,

$$K_{G5} = (v^2 - G_N M/r)/4G_N M = 9.90 \times 10^{-22} \text{ m}^{-1} \approx 10^{-21} \text{ m}^{-1}. \quad (276)$$

The second equation in Eq. (276) results from substituting in $v = 1.93 \times 10^5$ m/s at 80 kpc from Gnedin, *et al.*,²¹ $r = 80$ kpc $= 2.469 \times 10^{21}$ m, and $M = 1.279 \times 10^{41}$ kg from McMillan²² for the mass of the Milky Way. We also use the standard value of $G_N = 6.674 \times 10^{-11}$ m³/kg s².

The connection between the theory presented herein and observations is often the same as that given by presently prevailing theory, as we have derived Maxwell's equations, the Lorentz force equation, and Newtonian gravity. But now we see that our theory makes a new and specifically testable prediction for where dark matter should exist. One needs to simply calculate P_G from relevant massive bodies, then calculate $\delta\rho_{M1}$ from Eq. (262), and then integrate $\delta\rho_{M1}$ over the region of interest to find the amount of dark matter within that region.

D.7. Classic tests of general relativity

Any new theory of gravity must of course agree with the classic tests of general relativity just as well as general relativity does. The first classic test of general relativity involves the gravitational redshift. To calculate it from the quantum luminiferous aether we begin with Eq. (250), and we then set one mass equal to a stellar mass and the other mass equal to

the energy of a photon (hf) divided by c^2 . By integrating the force $G_N M_{\text{STAR}} hf/c^2 r^2$ over the distance traveled, we can find the energy change as the photon moves radially outward from the surface (at R_0) of the star: $E_{\text{PHOTON}}(r) - E_{\text{PHOTON}}(R_0) = G_N M_{\text{STAR}} hf/c^2 r - G_N M_{\text{STAR}} hf/c^2 R_0$. We then assign the photon energy at $r = \text{infinity}$ to hf and obtain the Newtonian limit for the redshift $E_{\text{PHOTON}}(R_0) = hf + G_N M_{\text{STAR}} hf/R_0 c^2 = hf(1 + G_N M_{\text{STAR}}/R_0 c^2)$.

The second classic test of general relativity involves advances of the perihelia. To calculate those advances, we numerically integrated Eq. (275). Results of the numerical integration are presented in Table I.

To achieve the results in Table I, we used the value for K_{G5} from Eq. (276) and then we set K_{G6} so that it obtained a good fit to the data

$$K_{G6} = 8.9167 \times 10^{-28} \text{ m/kg}. \quad (293)$$

Table I refers to work done by Pitjeva²³ who included over 250 parameters in a very complex numerical calculation. On the other hand, we have only used a very simple two-body calculation to obtain our results. Nonetheless, our simplified results are substantially in agreement with Pitjeva and general relativity.

The third classic test of general relativity involves the bending of light. It is well known that light bends in gravitational fields two times more than a naïve Newtonian approach would suggest.²⁴ Up to this point in our development, we have not yet proposed a gravitational flow law, but there must of course be one and so we propose:

The extrinsic-energy flow speculation. When extrinsic-energy flows through the aether, the tension in the positive (negative) attached-aether is reduced (increased) in the directions perpendicular to the flow with the magnitude of reduction (increase) proportional to the flow.

The above assertion is called a speculation rather than a law since more than one speculation may work to achieve the empirical result.

Light bending is treated in full detail in Ref. 1 and here we will just give an overview. The gravitational flow speculation results in a change to the force due to tension that is different in Z than in X and Y :

$$F_{\text{TPZ}} = K_{\text{TP}} X_Q = K_{\text{T0}} (1 - K_{\text{G1}} \rho_E) X_Q, \quad (294)$$

$$F_{\text{TNZ}} = K_{\text{TN}} X_Q = K_{\text{T0}} (1 - K_{\text{G2}} \rho_E) X_Q, \quad (295)$$

$$F_{\text{TPX}} = F_{\text{TPY}} = K_{\text{T0}} (1 - K_{\text{G1}} \rho_E) X_Q - K_{\text{T0}} K_{\text{F4}} \rho_{\text{FE}} X_Q, \quad (296)$$

$$F_{\text{TNX}} = F_{\text{TNY}} = K_{\text{T0}} (1 - K_{\text{G2}} \rho_E) X_Q + K_{\text{T0}} K_{\text{F4}} \rho_{\text{FE}} X_Q. \quad (297)$$

The forces of Eqs. (294)–(297) lead to energies and cube distortions within the analytic-cubes, and this is what leads to light bending that is a factor of two greater than the Newtonian amount. The light bending analysis in Ref. 1 is similar to what we have done with the Coulomb force and Newtonian Gravity. Slices and strips within a spherical shell are made, and then individual cubes within the strips are

TABLE I. Competing calculations of “anomalous” Perihelion advances.

Planet	Radius a (10^6 km)	Period T (days)	Pitjeva	Eq. (275) W/O K_{G5}	Eq. (275) With K_{G5}
Mercury	57.9	88.0	42.976 ± 0.005	42.977	42.977
Venus	108.2	224.7	8.644 ± 0.033	8.623	8.589
Earth	149.6	365.2	3.846 ± 0.007	3.837	3.804
Mars	227.9	687.0	1.343 ± 0.007	1.351	1.322
Jupiter	778.6	4331	0.067 ± 0.093	0.0622	0.0453
Saturn	1433.5	10 747	-0.010 ± 0.015	0.0135	0.001 04
Uranus	2872.5	30 589	-3.89 ± 3.90	0.002 38	-0.006 46
Neptune	4495.1	59 800	-4.44 ± 5.40	7.75×10^{-4}	-0.006 30
Pluto	5906.4	90 560	2.84 ± 4.51	4.17×10^{-4}	-0.005 65

evaluated. The energy of each cell is calculated both with and without a virtual displacement. The force on the sphere is found by dividing the energy difference caused by the virtual displacement by the amount of that virtual displacement. The resulting force leads to the empirical light bending formula (again, see Ref. 1 for details).

While not one of the original tests of general relativity, the Shapiro effect²⁵ is often called the fourth classic test. This test involves the changing light speed as light goes past a massive object such as the sun. Recall now Eq. (107), $T_0 = m_0 c^2$. Throughout our evaluations, we have seen that various aetherial parameters can vary depending upon conditions, and so we modify Eq. (107) to

$$V_{\text{LIGHT}} = (T/m)^{1/2}. \tag{342}$$

In Eq. (342), T and m may vary from their nominal values. In Ref. 1, we derive

$$T_P = (K_{T0}/X_0)[1 - 2K_c|P_G|/\xi_0 - 4K_c K_{GC}|P_G|/\xi_0], \tag{343}$$

$$T_N = (K_{T0}/X_0)[1 - 2K_c|N_G|/\xi_0 - 4K_c K_{GC}|N_G|/\xi_0]. \tag{344}$$

We then go on to propose:

The aetherial inertial-mass assignment. The ethereal inertial-mass density equals the field energy density plus the gravitational potential energy density divided by c^2 .

Now recall Eq. (22), $K_{T0}X_0^2 = 2K_{Q0}/X_0^2$, Eq. (11), $E_T = (1/2)K_{T0}X_0^2$, and Eq. (12), $E_Q = K_{Q0}/X_0^2$. We see that the sum of the quantum energy and tension energy is $E_Q + E_T = K_{T0}X_0^2$. Recall again Eq. (107), $T_0 = m_0 c^2$, and also recall Eq. (15) $F_{T0} = K_{T0}X_Q$. T_0 is the tension force per unit area, $T_0 = K_{T0}X_Q/X_0^2 = K_{T0}/X_0$ and $m_0 c^2$ is the equivalent mass-energy per unit volume which is $(E_Q + E_T)/X_0^3 = K_{T0}X_0^2/X_0^3 = K_{T0}/X_0$. Therefore, we see that Eq. (107) simply results from the leading term of the field energy.

Now the gravitational potential energy is $E_{P\phi} = E_{N\phi} = \rho_0 \phi$, where ρ_0 is the aetherial density and ϕ is the gravitational potential. From Eq. (246), $\phi_{SO} = \rho_{GS}R_S^3/3r\epsilon_0 - \rho_{GS}R_S^2/2\epsilon_0$ and resetting the arbitrary constant so that ϕ is zero at infinity we get a potential of $\phi_{SO} = \rho_{GS}R_S^3/3r\epsilon_0$. Using Eq. (238), $\rho_{GS} = 3[K_{G1} - K_{G2}]\rho_{ES}\rho_0/2 = 9[K_{G1} - K_{G2}]\gamma M_S c^2 \rho_0/8\pi R_S^3$, we get $\phi = 3[K_{G1} - K_{G2}]\gamma M_S c^2 \rho_0/8\pi\epsilon_0 r$

$= K_\phi/\rho_0 r$, where we have defined $K_\phi = 3[K_{G1} - K_{G2}]\gamma M_S c^2 \rho_0^2/8\pi\epsilon_0$. This leaves

$$E_{P\phi} = E_{N\phi} = \rho_0 \phi = K_\phi/r. \tag{347}$$

By our inertial mass assignment, using Eqs. (225) and (347), we get

$$m = \left[(K_{T0}/X_0)(1 - 4K_c^2 K_{GC}|P_G|^2/\xi_0^2) + K_\phi/r \right] / c^2. \tag{348}$$

We can now substitute Eqs. (343) and (348) into Eq. (342) to form the expression for the speed of light as it passes near the sun

$$V_{\text{LIGHT}} = (T/m)^{1/2} = c \{ (K_{T0}/X_0)[1 - 2K_c|P_G|/\xi_0 - 4K_c K_{GC}|P_G|/\xi_0] / [(K_{T0}/X_0)(1 - 4K_c^2 K_{GC}|P_G|^2/\xi_0^2) + K_\phi/r] \}^{1/2}. \tag{349}$$

Near the sun we assume $K_{T0}/X_0 \gg K_\phi/r \gg (K_{T0}/X_0)2K_c|P_G|/\xi_0 \gg (K_{T0}/X_0)4K_c^2 K_{GC}|P_G|^2/\xi_0^2$ and

$$V_{\text{LIGHT}} \approx c [1 - K_\phi/2(K_{T0}/X_0)r]. \tag{350}$$

In Fig. 10, M is the planet Mercury, E is the earth, and S is the sun. We will assume that the deflection of light by the sun is small enough that we can consider the path to be predominantly along the x axis of the figure. We then have $dx/dt = V_{\text{LIGHT}} \approx c[1 - K_\phi/2(K_{T0}/X_0)r]$, which for a small incremental spatial advance dx we can rearrange to $dt \approx (dx/c)[1 + K_\phi/2(K_{T0}/X_0)r]$. From this, and noting that $r = (x^2 + R_{\text{MIN}}^2)^{1/2}$ all along the path, the time for light to go from Mercury to Earth is $t = \int dt = (1/c) \int [1 + K_\phi/2(K_{T0}/X_0)(x^2 + R_{\text{MIN}}^2)^{1/2}] dx$. Evaluating the integral,

$$T_{\text{ME}} = x/c + (K_\phi X_0/2cK_{T0}) \ln \left[(x^2 + R_{\text{MIN}}^2)^{1/2} + x \right]. \tag{351}$$

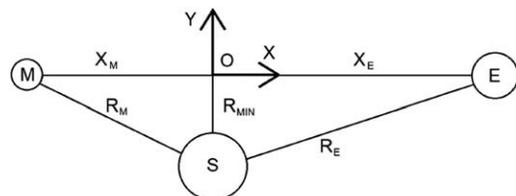


FIG. 10. Geometry of relevance for calculating the Shapiro effect.

We now evaluate the delay time of Eq. (351) between $-X_M$ and X_E ,

$$T_{\text{DELAY}} = (K_\phi X_0 / 2cK_{T0}) \ln [(R_E + X_E) / (R_M - X_M)]. \quad (352)$$

When constants are fit, Eq. (352) is the same equation as that given by general relativity in the low field limit, which is in agreement with experimental data for all values of R_{MIN} .

While not a classic test of general relativity, gravitational waves are also important. Recall that for electromagnetism we have derived Eq. (104), $m_0(\partial^2 \mathbf{P} / \partial t^2) = T_0 \nabla^2 \mathbf{P} - K_{F3} \mathbf{J}_T$, where $m_0(\partial^2 \mathbf{P} / \partial t^2)$ is the inertial-mass density multiplied by the acceleration, $T_0 \nabla^2 \mathbf{P}$ is the tension force, and $K_{F3} \mathbf{J}_T$ is the detached-aether flow force. Following the same reasoning and derivation, for gravitational disturbances we get $m_0(\partial^2 \mathbf{P}_G / \partial t^2) = T_0 \nabla^2 \mathbf{P}_G + \mathbf{F}_{\text{GFP}}$, where \mathbf{F}_{GFP} is the extrinsic-energy flow force on the positive-attached-aether. In both the electromagnetic and gravity cases, we will get waves upon the aether in regions where $K_{F3} \mathbf{J}_T$ and \mathbf{F}_{GFP} are zero, respectively. And since the mass and tension densities are the same aetherial attributes for both cases, we see that gravity waves should move at the same speed that light waves do.

PART E. DENSE STELLAR OBJECTS

Unlike the theory of general relativity, we have no singularity and no concept of a ‘‘black hole.’’ However, since gravitational fields of dense, ultramassive objects have been observed near the center of many galaxies, it is important to have some modeling of what these objects might be.

Reference 1 discusses dense stellar objects in an appendix since it does not contain the rigor of the main paper. The goal of the study is to obtain a simple modeling for dense stellar objects that achieves a rough quantitative agreement with the data so that the model can explore the effects that field-mass has on those objects. An approach to modeling of dense stellar objects is proposed that involves analyzing small cubes of matter within white dwarfs and neutron stars, rather than treating the whole star as a single degenerate quantum system. The argument is made that collisions between the fermions of a dense star will lead to a collapse of the fermion’s wave function, and these collapses therefore form cubic square-well boundaries. Within such square-

wells, many fermions can be in degenerate states, but far less than for an entire star. The analysis then develops an expression for the quantum-pressure at the walls of the cube, and this can then be used along with the equation of hydrostatic equilibrium to develop numerical integration programs to calculate the mass, radius and density profile of dense stellar objects. By including an empirical adjustment factor of 1.2, those numerical programs give an excellent match to present observations for the case of white dwarfs. The factor of 1.2 is reasonable, since we neglect the effects of fusion, internal Coulomb interactions, star rotation, and thermal effects in our modeling.

Our numerical model is then used to study the effects of the field-masses. It is determined that the first-field-mass is not significant. However, the second-field-mass can become important. Recall Eq. (271), $\Delta M_{2\text{OUT}} = 5K_{G6}(\gamma_M M)^2 / r - 6K_{G6}(\gamma_M M)^2 / R$, for $r > R$. Evaluating Eq. (271) at the edge of the sphere shows that $\Delta M_{2\text{OUT}} = -K_{G6}(\gamma_M M)^2 / R$ when $r = R$. Hence, when M is large and R is small, the second-field-mass may become quite large. In Ref. 1, we find that the second-field-mass is of the order of 0.1% of the stellar mass for typical white dwarf masses.

For neutron stars, the mass density gets much larger and so does the effect of the negative second-field-mass. Table J11 presents results for a simplistic numerical evaluation of various neutron stars. All numerical calculations begin at the center of the star and integrate outward. By increasing the assumed starting density different masses and radii are predicted for the neutron stars. It is seen that the negative second-field-mass becomes very nearly equal to the normal mass as the neutron stars become more massive. Also, it is seen toward the bottom of the table that even with enormous central densities, the observed net mass remains quite limited.

Of course, it may be possible that something changes inside of neutron stars when pressures get very high. Two possibilities come to mind quickly: (1) we can speculate that very high pressure will force matter into a state made up of exotic particles; and/or (2) we can speculate that the gravitational force equation becomes altered as pressures become very high. The appendix in Ref. 1 evaluates both of these speculations. Crushing the neutrons into a single new type of exotic matter does not fit observations. (It is possible that crushing matter into several different types of exotic matter over different pressure regions would fit observations, but

TABLE J11. Results of simplistic numerical integrations for a neutron star, various ρ_{WD0} values.

Evaluation point	ρ_{WD0} (kg/m ³)	Radius (m)	Positive mass (Ms)	Negative mass (Ms)	Net mass (Ms)
Neutron-degeneracy/free space boundary	1×10^{18}	17 209	1.3191	0.4414	0.8777
~10 times greater than the electron-degeneracy/free space boundary	1×10^{18}	173 810	1.3191	0.6980	0.6211
Neutron-degeneracy/free space boundary	1×10^{19}	16 234	5.6251	3.8744	1.7508
~10 times greater than the electron-degeneracy/free space boundary	1×10^{19}	163 963	5.6251	4.6975	0.9277
Neutron-degeneracy/free space boundary	1×10^{21}	19 600	99.455	95.285	4.169
10 times greater than the electron-degeneracy/free space boundary	1×10^{21}	196 000	99.455	97.962	1.493
Neutron-degeneracy/free space boundary	1×10^{24}	22 300	2274	2268	6.19
10 times greater than the electron-degeneracy/free space boundary	1×10^{24}	223 000	2274	2272	1.84
Neutron-degeneracy/free space boundary	1×10^{27}	26 009	118 571	118562	9.20
~10 times greater than the electron-degeneracy/free space boundary	1×10^{27}	262 692	118 571	118569	2.26

TABLE J13. Results of numerical integration assuming a saturation effect for various initial densities, gMult = 1.

Description	$\rho_{\text{NSO}} \text{ (kg/m}^3\text{)}$	Stellar radius (m)	Positive mass (Ms)	Negative mass (Ms)	Net mass (Ms)
0.75 Ms N-Star	4.1×10^{17}	20 877	1.345	0.5901	0.7548
1.9 Ms N-Star	5.0×10^{17}	30 639	4.765	2.861	1.904
46 Ms N-Star	8.0×10^{17}	244 681	2422	2377	45.92
332 Ms N-Star	1.0×10^{18}	1.511×10^6	570 184	569 852	331.8
Sgr A*	1.62×10^{18}	2.297×10^9	2.005×10^{15}	2.005×10^{15}	3.807×10^6
Andromeda central star	1.74×10^{18}	1.220×10^{10}	3.001×10^{17}	3.001×10^{17}	9.076×10^7
TON 618	1.96×10^{18}	3.138×10^{11}	5.110×10^{21}	5.110×10^{21}	5.758×10^{10}

this speculation was not studied.) However, a simple proposal that P_G and N_G saturate at high pressures does fit observed data, as shown in Tables J13–J15. (The gMult factor is related to the point where saturation occurs.)

An ultra-massive object named Sgr A* is observed at the center of the Milky Way galaxy. A recent measurement²⁶ gives a diameter of 51.8 $\mu\text{arc s}$ (6.18×10^{10} m) for the central ring around Sgr A*, and it is seen that the radii calculated for Sgr A* in Tables J13–J15 are within that value. However, one aspect of the results shown in Tables J13–J15 is quite significant, and that is the amount of negative second-field-mass predicted to exist within supermassive objects. The tables indicate that the vast majority of the hadronic mass is cancelled out by the second-field-mass in supermassive neutron stars. Of course, the model used to produce Tables J13–J15 is only speculative and other models are certainly possible.

PART F. CLOSING COMMENTS

F.1. Present problems of physics are addressed

Above we see that the quantum luminiferous aether addresses many of the problems of present physics. By stepping away from the point-like theory of relativity, the infinities associated with points are no longer a fundamental issue. We have presented a speculative alternative to one such infinity, as we propose a saturation effect that leads to supermassive neutron stars as an alternative to relativity’s black holes. There is no longer a cosmological constant to be concerned with. Dark matter is understood. And by returning to a flat and Euclidean space, absolute time, and absolute simultaneity, we can easily understand quantum mechanics as an instantaneous collapse of wave functions. (The concept of “instantaneous” is once again valid.)

F.2. An understandable physical model

Beyond addressing problems that are acknowledged by the present physics community, the model presented herein is also a return to a physical model of our world, in contrast to the present emphasis on mere mathematical models. This

makes physics understandable once again. Equations simply accepted as “nature’s laws,” which include Maxwell’s equations, the Lorentz force equation, and Newton’s equation of universal gravitation, are all found to have a common physical underpinning.

To see some of the physical interpretations that the theory herein enables, we begin by recalling Poisson’s equation, Eq. (55), $\nabla^2\phi = -\rho_D/\epsilon_0$. In Eq. (55) and elsewhere herein, ρ_D is identified as free, detached-aether. Conventionally ρ_D is identified as electric charge. From this observation, we see that electric charge is an amount of aether that has become detached from the predominant attached-aether. Positive-aether includes positive electric charge and negative-aether is negative electric charge. This identification shows that the positive (and separately, the negative) attached-aether includes an infinite sea of charge, which is internally attached and in the state of a solid. And since electric current is understood as the motion of electric charge, electric currents are now identified as moving detached-aether.

Physical interpretations can also be found for the static electric field, the vector potential, and light. The scalar potential ϕ satisfies Eq. (56), $\mathbf{P}_L - \mathbf{N}_L = \nabla(\psi_P - \psi_N) = -\epsilon_0\nabla\phi/\rho_0$. With the static electric field $\mathbf{E} = -\nabla\phi$ from Eq. (123), this reveals that the static electric field is simply proportional to the longitudinal aetherial separation $\mathbf{P}_L - \mathbf{N}_L$. Next, consider Eq. (114), $\mathbf{P}_T = -\mathbf{N}_T = -K_{F3}\mathbf{A}/\mu_0T_0$. We see that the vector potential \mathbf{A} is proportional to the transverse displacement of the aether from its nominal position. And of course, light is now identified as an aetherial wave, just as the classical theorists anticipated.

F.3. Topics for future consideration

This paper has presented the theoretical foundations for the quantum luminiferous aether, and while it accomplishes much, there are several areas where future work can and should be done. One area for future study involves doing a deeper analysis. In this work we have shown that Maxwell’s equations, the Lorentz force equation, and the equations of gravity all result from an aetherial model wherein we have

TABLE J14. Results of numerical integration assuming a saturation effect for various initial densities, gMult = 0.3.

Description	$\rho_{\text{NSO}} \text{ (kg/m}^3\text{)}$	Stellar radius (m)	Positive mass (Ms)	Negative mass (Ms)	Net mass (Ms)
1.0 Ms N-Star	1.05×10^{17}	33 757	1.405	0.4199	0.9849
Sgr A*	3.6×10^{17}	2.778×10^9	7.823×10^{14}	7.823×10^{14}	4.230×10^6
TON 618	4.35×10^{17}	3.977×10^{11}	2.294×10^{21}	2.294×10^{21}	6.184×10^{10}

TABLE J15. Results of numerical integration assuming a saturation effect for various initial densities, gMult = 0.1.

Description	ρ_{NSO} (kg/m ³)	Stellar radius (m)	Positive mass (Ms)	Negative mass (Ms)	Net mass (Ms)
1.1 Ms N-Star	3.0×10^{16}	52 279	1.362	0.261	1.101
Sgr A*	9.41×10^{16}	3.352×10^9	3.589×10^{14}	3.589×10^{14}	3.677×10^6
TON 618	1.14×10^{17}	5.396×10^{11}	1.497×10^{21}	1.497×10^{21}	6.779×10^{10}

kept terms to first order in non-vanishing quantities. Future efforts may involve analyses that include the discarded terms.

Future efforts may also involve more analysis and observation regarding the aetherial quantum itself. Nothing in the theory presently determines the size of the aetherial quantum: the aetherial quantum size ζ_Q is one of only three free parameters for our theory, the other two being the coupling parameters K_{GC} and K_c (see Ref. 1). Since Maxwell's equations coupled with quantum mechanics gives such an excellent treatment of the hydrogen atom, we can speculate that ζ_Q is sub-atomic, but more work needs to be done concerning a better determination of ζ_Q .

A reviewer of this work mentioned that the cubic nature of the quanta shown in Fig. 1 may imply that the aether has an underlying lattice structure, and that this might lead to artifacts that can be observed experimentally. If each aetherial quantum is in its own cubic potential well, then such artifacts may indeed be found. However, there is another possibility. If we postulate that quantum states collapse only when a momentum transfer occurs, then the boundary of the bounding cubic square-well will be determined only when momentum transfers are robust enough to define the square-well boundaries. In this case, this larger cubic square-well may contain many aetherial quanta within a degenerate quantum system. And in this case, the quantum shown in Fig. 1 should be viewed as a representative quantum, with properties set so that a collection of such representative cubes yields the same tension, quantum, delta, and gamma force fields as what are found at the boundaries of the larger well. In the latter case, no artifacts of a lattice are expected. (In Ref. 1, a representative quantum cube is used to analyze an electron degenerate system in white dwarfs; please refer to Ref. 1 for details on this analytic approach, as this approach can be applied to the aetherial quanta as well.) In either interpretation of Fig. 1 (a single quantum in its own cubic square-well or a representative quantum within a larger cubic square-well), the derivations of the fundamental physics equations follow. The question for further research is which of these two interpretations best represents nature.

Another reviewer of this work raised the issue as to why positive-detached-aether is predominantly contained within protons while negative-detached-aether is predominantly contained within much lighter electrons, and under what con-

ditions detached-aether might occur elsewhere. Of course, negative-detached-aether also occurs in anti-protons, while positive-detached-aether occurs in positrons, and there is a large particle zoo of other particles that contain electric charge. Why the detached-aether forms within certain specific particles is a question for future research.

Other topics for future studies include those related to cosmology such as dark energy, the big bang, gravitational lensing by galaxies, anisotropies in the cosmic microwave background, inflation, and the baryonic Tully–Fisher relation. While all of these topics are important, they predominantly involve observations from very far away. Note that the observations themselves are presently interpreted by assuming that general relativity is correct, and this then leads to certain conclusions which may no longer be accurate once when we consider them under the aether-based theory described herein.

¹See the supplemental material at <https://larsonism.com/Aether.htm> for a more detailed version of this work.

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